

People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research.
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Faculty of Natural and Life Sciences

Handout
Biophysics

Courses and corrected exercises
2nd year Licence, Faculty of Natural and Life Sciences

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Foreword

The general aim of the biophysics course is to enable students in the natural and life sciences to acquire a basic understanding of physics for the various applications in biology, which is a science of the living environment.

In physics, the observation of a phenomenon is followed by reasoning to write down the law of the phenomenon studied. This important approach deserves to be developed in all fields of science, particularly in biology and the medical sciences. Biophysics is an important subject for 2nd year and Masters students. It's a selective subject, as it's one of the few disciplines that requires real thinking. The aim of this handout is to present the various physical phenomena relevant to biology. This handout is spread over several chapters in each chapter we present a summary of the course followed by examples and exercises that we covered during the 2018-2019 year in the three biology pathways (biological sciences, food sciences and biotechnologies) in the biology department. This course handout also presents a summary of the courses we taught between 1998-2019, initially as a lecturer and then as a module leader in the Department of Natural and Life Sciences at the Université d'Oran 1 Ahmed Ben Bella.

I would also like to thank the active biophysics team, N.Touhami, N.Guettari, A.Bechlaghem and H.Benmaaza for their fruitful participation in the creation of the tutorial sheets for the 2018/2019 year.

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Reminder: Dimensional analysis

1-Introduction :

The measurement of any physical quantity is expressed with a unit. A distinction is made between fundamental units, which are used to create a coherent system of units, and derived units, which can be expressed in relation to the fundamental units.

Example: the International System (SI) is based on the meter, kilogram, second and ampere, and has been called M.K.S.A (meter, kilogram, second, ampere), the kelvin and a few other units in specialized fields. The C.G.S. system (centimeter, gram and second) is still used in some countries.

2- The basic units of the international system (S.I) :

Quantity	Unit	Symbol	Designation	Formula and dimension
Length	Meter	M	L, l	$[l]=L$
Mass	Kilogram	Kg	m,M	$[m]=M$
Time	Second	S	T, τ , t	$[t]=T$
Electrical current	Ampere	A	i, I	$[i]=I$
Thermodynamic temperature	Kelvin	K	T	
Quantity of matter	Mol		n	
Luminous intensity	Candela	Cd	I	

Additional units :

Radian (rd) for flat angle, steradian (sr) for solid angle.

3- The derived units:

A few mechanical and electrical units are shown here in the form of a table.

Physical quantity	Formula and dimension	Expression in basic unit	Unit name and symbol
Area (S)	$[S] = [l^2] = L^2$	m^2	
Volume (V)	$[V] = [l^3] = L^3$	m^3	
Speed (v)	$[v] = \frac{[l]}{[t]} = L.T^{-1}$	$m.s^{-1}$	
Force (F)	$[F] = [m.a] = [m].[a]$ $= MLT^{-2}$	$Kg.m.s^{-2}$	Newton (N)
Acceleration (a)	$[a] = \frac{[v]}{[t]} = LT^{-1} / T$ $= LT^{-2}$	$m.s^{-2}$	

Work, energy	$[w] = [F] \cdot [l]$ $= M \cdot L^2 \cdot T^{-2}$	$Kg \cdot m^2 \cdot s^{-2}$	Joule (j)
Power	$[p] = \frac{[w]}{[t]} = M L^2 T^{-2} / T$ $= M L^2 T^{-3}$	$Kg \cdot m^2 \cdot s^{-3}$	Watt(w)
Electric charge	$[q] = [i \cdot t]$ $= I T$	A.S	Coulomb (C)
Potential difference	$[v] = \frac{[p]}{[i]} = M L^2 T^{-3} / I$	$Kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$	Volts (v)
Resistance	$[R] = \frac{[V]}{[i]} = M L^2 T^{-3} I^{-2}$	$Kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$	Ohm (Ω)

4- Physical constants in SI:

Constant	symbol	Most precise value
Perfect gas constant	R	8.3143 J. K ⁻¹ .mol ⁻¹ unit in (SI) When pressure is expressed in atmosphere atm and volume in liter, the value of R is R= 0.082 atm.l.mol ⁻¹ .k ⁻¹
Avogadro number	NA	6.02214 x 10 ²³ mol ⁻¹
Boltzmann constant	k _B	1.380648 x 10 ⁻²³ J K ⁻¹
Electric charge	e	1.6021 10 ⁻¹⁹ C

5- Formula consistency:

When we have a relationship of equality between quantities (for example, when solving an exercise) we can check the plausibility, or coherence, of this relationship, by verifying that the two dimensions on either side are identical.

Application :

Verification of the relationship: $E = m c^2$ between mass and energy

Energy = work = force x displacement

Hence $[E] = [F] \cdot [l] = M L T^{-2} \cdot L = M L^2 T^{-2}$

Usual prefixes and non-system units :

Factor	10 ¹²	10 ⁹	10 ⁶	10 ³	10 ⁻³	10 ⁻⁶	10 ⁻⁹	10 ⁻¹²
Prefix	Tera	Giga	Mega	Kilo	Milli	Micro	Nano	Pico
Symbol	T	G	M	K	m	μ	n	P

Name	Symbol	Value in SI
Angstrom	Å	1 Å = 10 ⁻¹⁰ m

Electron volt	eV	1eV=1.6 021910 ⁻¹⁹ joule
Calorie	Cal	1cal= 4.1867 joule
Normal atmosphere	Atm	1atm=1.01325 x 10 ⁵ Pa
Tower	mmHg	1mmHg=133.3 Pa
Bar	Bar	1bar= 10 ⁵ Pa
Poise	P	1poise = 0.1 Pa.s

The pascal times the second is the unit of dynamic viscosity in the International System. It is a derived unit; in terms of base units it is expressed as follows: $1 \text{ Pa s} = 1 \text{ kg m}^{-1} \text{ s}^{-1}$. The pascal times the second is called the poiseuille.

Exercises :

Exercise 1 :

Using simple formulas, establish the dimensions and fundamental units of the following quantities: velocity v , acceleration a , force F , surface S , volume V , density ρ , energy E , pressure P , charge q .

Exercise 2 :

From the definition formulas, determine the dimension and then the imity in the S.I. of the following physical constants:

1/ The perfect gas constant R defined by $PV= nRT$ where n is the number of moles of gas, T the temperature, P the pressure, V the volume.

2/ Boltzmann's constant k defined by the internal energy of a monoatomic perfect gas molecule at temperature T : $U = (3/2) kT$

3/ Avogadro's constant, where $k=R/N_A$, N_A is Avogadro's number.

4/ Faraday's constant F , defined by $F= N_A e$, where e is the elementary charge.

Exercise 3:

1/ The value of the frictional force exerted by a fluid on a sphere of radius r moving at low speed v relative to the fluid is given by Stokes' relation:

$F= 6\pi \eta r v$ where η is the viscosity of the fluid. Establish the dimensional equation of η .

2/ The unit of η in the CGS system is the poise. Its unit in the S.I. is the poiseuille.

Find the conversion coefficient between the two units?

Exercise 4:

1/ The value of the frictional force exerted by a fluid on a sphere of radius r moving at low speed v relative to the fluid is given by Stokes' relation:

$$[A]=M^{-2}L^2T^{-1}$$

This physical quantity is calculated from the equation: $A = \alpha v^2 F^2 + \beta p^{-1} m g^{-3}$

where F is a force, v a velocity, m a mass, p a pressure and g the acceleration of gravity.

State the dimension of α and β so that the equation is dimensionally correct.

Exercise 5:

The frequency of vibration of a drop of water can be written as :

$$f = k R^\alpha \rho^\beta \sigma^\gamma$$

where k is a dimensionless constant. R is the radius of the drop, ρ its density, σ is the surface tension defined by a force per unit length.

Use dimensional analysis to determine the values of the parameters α, β et γ .

Answers :

Exercise1 : $v = \frac{l}{t} \Rightarrow [v] = \frac{[l]}{[t]} = \frac{L}{T} = LT^{-1} (ms^{-1})$.

$$x = \frac{1}{2}at^2 \Rightarrow [a] = \frac{[x]}{[t^2]} = LT^{-2} (ms^{-2})$$

$$F = m \cdot a \Rightarrow [F] = [m] \cdot [a] = M \cdot LT^{-2} (kgms^{-2})$$

$$S = l^2 \Rightarrow [S] = [l] \cdot [l] = L^2(m^2)$$

$$V = l^3 \Rightarrow [V] = [l] \cdot [l] \cdot [l] = L^3(m^3)$$

$$f = \frac{m}{V} \Rightarrow [f] = \frac{[m]}{[v]} = ML^{-3}(kgm^{-3})$$

$$E = \frac{1}{2}mv^2 \Rightarrow [E] = [m] \cdot [v]^2 = M \cdot L^2 \cdot T^{-2}(kgm^2s^{-2}) \text{ or Joule}$$

$$P = \frac{F}{S} \Rightarrow [P] = \frac{[F]}{[S]} = \frac{M \cdot L \cdot T^{-2}}{L^2} = M \cdot L^{-1}T^{-2} (kgm^{-1}s^{-2})$$

$$Q = i \cdot t \Rightarrow [Q] = [i] \cdot [t] = I \cdot T (A \cdot s)$$

Exercice 2 :

$$PV = nRT$$

$$[R] = \frac{[P][V]}{[n][T]} = ML^{-1}T^{-2} \cdot \frac{L^3}{mol \theta} =$$

$$M L^2 T^{-2} mol^{-1} \theta^{-1} \text{ Unité } (kg m^2 s^{-2} mole^{-1} K^{-1}) \text{ ou bien } (J K^{-1}mol^{-1})$$

$$(R = 8,314 J K^{-1}mol^{-1})$$

$$\text{Or } R = 0,082 \text{ l atm mol}^{-1}K^{-1}$$

$$2) U = \frac{3}{2}kT$$

$$[k] = \frac{[U]}{[T]} = \frac{ML^2T^{-2}}{\theta} = ML^2T^{-2}\theta^{-1}(\text{Joule } K^{-1})$$

$$3) R = Na k$$

$$[Na] = \frac{[R]}{[k]} = \frac{ML^2T^{-2}mol^{-1}\theta^{-1}}{ML^2T^{-2}\theta^{-1}}$$

$$[Na] = mol^{-1}$$

$$4) [F] = [Na] \cdot [e] = mol^{-1} \cdot I \cdot T \quad (mol^{-1}A \cdot s)$$

Exercice 3 :

$$F = 6\pi nrv$$

$$[n] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{LLT^{-1}}$$

$$[n] = ML^{-1}T^{-1}$$

unité ($kg\ m^{-1}s^{-1}$ poiseuille)

CGS $cm\ g\ s$

$$1\ poiseuille = (10^3\ g\ 10^{-2}\ cm^{-1}s^{-1}) = 10\ g\ cm^{-1}\ s^{-1}$$

1poiseuille \rightarrow 10 poise Knowing that poise is $g\ cm^{-1}\ s^{-1}$

Exercice 4 :

$$[A] = M^{-2}L^2T^{-1}$$

$$\begin{aligned} [\alpha v^2 F^2] &= [\alpha] \cdot [v]^2 [F]^2 \\ &= [\alpha] \cdot (LT^{-1})^2 (MLT^{-2})^2 \\ &= [\alpha] L^2 T^{-2} M^2 L^2 T^{-4} \end{aligned}$$

$$M^{-2}L^2T^{-1} = [\alpha] \cdot L^4 T^{-6} M^2$$

$$[\alpha] = M^{-2}L^2T^{-1}L^{-4}T^6M^{-2}$$

$$[\alpha] = L^{-2}T^5M^{-4}$$

$$[A] = [\beta][P]^{-1}[m][g]^{-3}$$

$$\begin{aligned} M^{-2}L^2T^{-1} &= [\beta] \cdot (ML^{-1}T^{-2})^{-1} M[g]^{-3} \\ &= [\beta] \cdot M^{-1}LT^2ML^{-3}T^6 \end{aligned}$$

$$M^{-2}L^2T^{-1} = [\beta] \cdot L^{-2}T^8$$

$$[\beta] = M^{-2}L^2T^{-1}L^2T^{-8}$$

$$[\beta] = M^{-2}L^4T^{-9}$$

Chapter1 : The states of matter :

1- Introduction :

In matter, molecules are subject to two different abilities: the first is the ability to assemble, essentially due to intermolecular forces of attraction (Van der Waals forces) of the $F=K/r^7$ type, essentially electrostatic due to the presence of charges (ionization) or dipoles, or intermolecular forces of repulsion (Born or Paul forces) of the $F=K/r^{13}$ type, resulting in the entanglement of electronic clouds. The other ability is dispersion, due to the phenomenon of "thermal", disordered agitation (Brownian motion). Average kinetic energy is proportional to absolute temperature (the constant of proportionality is Boltzmann's constant $k = 1.38 \cdot 10^{-23}$). The total energy of a molecule is $E_T = E_L + E_{TRANS}$

The macroscopic behavior of a population of molecules depends essentially on the respective importance of binding (E_L) and translational (E_{TRANS}) energies.

Following the prevailing trend, we distinguish three fundamental physical states of matter: solid, liquid and gas.

If the binding energy $E_L \gg E_{TRANS}$, we have a solid state with no dispersion and therefore an ordered arrangement of atoms.

If the binding energy $E_L \ll E_C$ we have a gaseous state and therefore a dispersion of molecules with no shape specific to the fluid.

If the binding energy $E_L \sim E_C$ we have a liquid state no dispersion no shape proper to the fluid.

2- Gases:

The gaseous state is a dispersed state. It's perfect disorder. The distance between molecules is very great, so intermolecular bonds are often negligible. There are two types of gas: perfect and real.

2-1 Real gases :

Molecules are more concentrated (high pressure), occupying a non-negligible part of the available volume (possible interactions). It's a non-condensed state (non-coherent or dispersed), totally disordered, with no form of its own: it's a fluid.

2-2 Perfect gases :

Molecules are weakly concentrated (low pressure) and have small volumes relative to the available volume (molecules, no interactions between them).

2-2-1 Pressure of a perfect gas :

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The pressure of a perfect gas is due to the impact of the gas molecules on the walls of the container in which it is contained. It is proportional to temperature T and to the number of molecules per unit volume - Boyle Mariotte's law. $P V = n R T$

With R: the perfect gas constant = $8.314 \text{ J mol}^{-1}\text{K}^{-1}$, V: volume in m^3 , T: temperature in K, n: number of moles, P: pressure in pascals.

Let's consider two gases G1 and G2 respectively with two concentrations n_1 and n_2 .

For gas G1 we have $P_1 = \frac{n_1}{V} RT$; for gas G2 we also have $P_2 = \frac{n_2}{V} RT$

The pressures P_1 and P_2 exerted by the gas molecules G_1 and G_2 respectively are called partial pressures.

A perfect gas mixture is also a perfect gas.

Consider P the total pressure of the mixture, The pressure exerted by a mixture of multiple perfect gases is determined by adding the partial pressures of each of the constituent gases in the mixture.

$$P = \sum_{i=1}^N P_i$$

P: is the total pressure of N perfect gases, and p_i is the partial pressure of gas G_i

Find the relationship between partial pressure and total pressure.

2-2-2 Mixtures of perfect gases :

Consider two gases G_1 and G_2 respectively with two different concentrations n_1 and n_2 . P: the total pressure of the gas and n: the total mole number of the gas.

We have $P_1 = \frac{n_1}{V} RT$; $P = \frac{n}{V} RT$

From these two equations we can write that

If we have a mixture of several perfect gases, we have

$$P_i = \frac{n_i}{n_{total}} P_T$$

Where n_i = number of moles of gas G_i

n_{total} = number of moles of mixture $n = \sum_{i=1}^N n_i$

P: mixture pressure

Pi= partial pressure of the gas.

The mole fraction of the gas is defined as $X_i = \frac{n_i}{n_{total}}$

2-2-3 Mean kinetic energy and quadratic velocity of a perfect gas molecule :

According to the kinetic theory of perfect gases, the pressure exerted by a gas is due to the impact of the molecules on the container wall. The absolute temperature is also proportional to the mean kinetic energy of the molecules, hence :

$$\bar{E} = \frac{3}{2} k_B T. \text{ with } k_B: \text{ Boltzmann constant } = 1.38 \cdot 10^{-23} \text{ J/K.}$$

N : Avogadro number = $6.022 \times 10^{23} \text{ mole}^{-1}$.

If we consider N molecules of a GP $\bar{E}_c = \frac{1}{2} m \bar{V}^2$.

The mean square velocity is the following quantity :

$$V_{qm} = \sqrt{\overline{V^2}}, \text{ so we have } \bar{E}_c = \frac{1}{2} m V_{qm}^2.$$

We can write $\frac{1}{2} m V_{qm}^2 = \frac{3}{2} k T$.

$$V_{qm} = \sqrt{\frac{3 k T}{m}}$$

With $M = N m \Rightarrow m = \frac{M}{N}$ and we also have $R = N k_B$

$$V_{qm} = \sqrt{\frac{3 k_B T N}{M}} = \sqrt{\frac{3 R T}{M}}$$

With M: molar mass in Kg/mol.

Application:

Consider spherical particles with a diameter of $0.5 \mu\text{m}$ and a density of 1 g/cm^3 . Calculate the root-mean-square velocity of these particles at room temperature 20°C . Can the motion of these particles be observed under a microscope?

The data are: molecule diameters $d=0.5 \cdot 10^{-6} \text{ m}$, $\rho=1 \text{ g/cm}^3=1000 \text{ kg/m}^3$, $T=293 \text{ K}$.

$$V_{qm} = \sqrt{\frac{3 R T}{M}}$$

$$M = m N = \rho V N = \rho \frac{4\pi}{3} r^3 N$$

$$V_{qm} = \sqrt{\frac{3RT}{\rho \frac{4\pi}{3} r^3 N}} = \sqrt{\frac{3 \cdot 8.314 \cdot 293}{1000 \frac{4\pi}{3} (0.25 \times 10^{-6})^3 \cdot 6.022 \times 10^{23}}}$$

$$V_{qm} = 0.0136 \text{ m/s}$$

this makes microscopic observation possible.

2-2-4 Total energy of a perfect gas :

The internal energy of a perfect gas $u = E_{ctotale} = N E_c = N \frac{3}{2} k_B T$

N: is the number of GP molecules

If there are n moles of GP, then N : Avogadro number

$$N = N n, R = N k_B$$

$$u = N \frac{3}{2} k_B T = N n \frac{3}{2} k_B T = \frac{3}{2} n RT = \frac{3}{2} P V$$

2-2-5 Change of state :

If a solid is heated ($E_L \gg E_{Trans}$), E_{Trans} increases progressively and becomes of the same order of magnitude as E_L , with transition to the liquid state, then, if further heating is applied, transition to the gaseous state.

$E_{Trans} \gg E_L$.

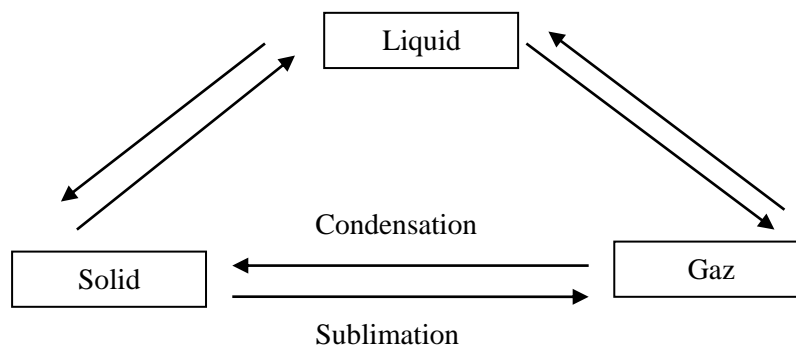


Figure1.1 The different states of matter and how they change.

3-Liquids :

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The liquid state is an intermediate state between the gaseous and solid states. It has properties in common with each of them

- 1- It's a fluid, like gases
- 2- It is a condensed state like solids.

Since intermolecular distances are very small, this has important consequences for liquids:

a- Liquids have a well-defined volume, since molecules attract each other sufficiently to remain close to each other.

b- Consequently, cohesion is not very important, since the liquid takes the shape of the container it occupies.

c- The liquid is characterized by a large volume.

d- Molecules are almost contiguous, so liquids have low compressibility.

3-1 Structure and properties of water :

The water molecule H_2O is made up of two OH bonds 0.96\AA long, making an angle of 104.28° between them. This asymmetrical structure, with an excess of negative charge on the oxygen and positive charge on the hydrogen, gives the water molecule a high dipole moment of 1.84 debye (1 debye = $1D = 3.336 \cdot 10^{-30} \text{ C m}$).

The positive charges carried by the hydrogens in water cause electrostatic bonds of a particular type with the electronegative atoms of neighboring molecules. These are called hydrogen bonds, and they are at the origin of the physicochemical properties of water, starting with the associations of water molecules with each other.

- In its liquid state, water has a pseudocrystalline structure, with each water molecule linked to four neighboring molecules.

- In its solid state, ice has an even more organized hexagonal structure.

Temperature is defined on a scale of degrees Celsius between 0°C and 100°C .

0°C : Freezing point of water.

100°C : Boiling point of water at normal atmospheric pressure.

3-2 Dissolution :

When a **solid** forms a homogeneous mixture with water, we say that :

- The solid dissolves in water.

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- The solid is soluble in water.

Examples: salt and sugar are soluble in water.

The mixture obtained during dissolution is called an aqueous solution.

The water used to dissolve the solid acts as a solvent.

The dissolved solid acts as the **solute**.

4-Solid :

The solid state corresponds to a crystalline structure whose elements (atoms, ions or molecules) are ordered periodically in space.

Example: a centered cubic crystal.

Depending on the physical nature of the elements placed at the nodes of the crystal, we distinguish 3 types of crystals

1- Ionic crystals, e.g. sodium chloride NaCl, and more generally electrolytes, as the crystal is made up of Na^+ and Cl^- ions.

2- Atomic crystals, e.g. diamond C

3- Molecular crystals, such as benzene.

In a crystalline state, atoms, ions or molecules remain in a specific place. The only disorder factor is the vibration or oscillation of atoms around their equilibrium position (thermal agitation). In molecular crystals, the molecules rotate in place.

If the solid is heated, thermal agitation increases, and when the vibration energy is sufficient, the cohesive forces that ensure the regularity of the crystalline structure are negligible. The solid state then changes to the liquid state, and we have the phenomenon of melting, which occurs at a specific melting temperature. It should be noted that molecular crystals have a lower melting temperature than atomic crystals, as their cohesive forces are even weaker.

Intermediate states :

5-1- Glasses :

A non-crystalline solid exhibiting the glass transition phenomenon. Rapid cooling of a liquid below T_{melt} produces a supercooled liquid (metastable state) and then a glass (non-

equilibrium state) below T_g , the glass transition temperature.

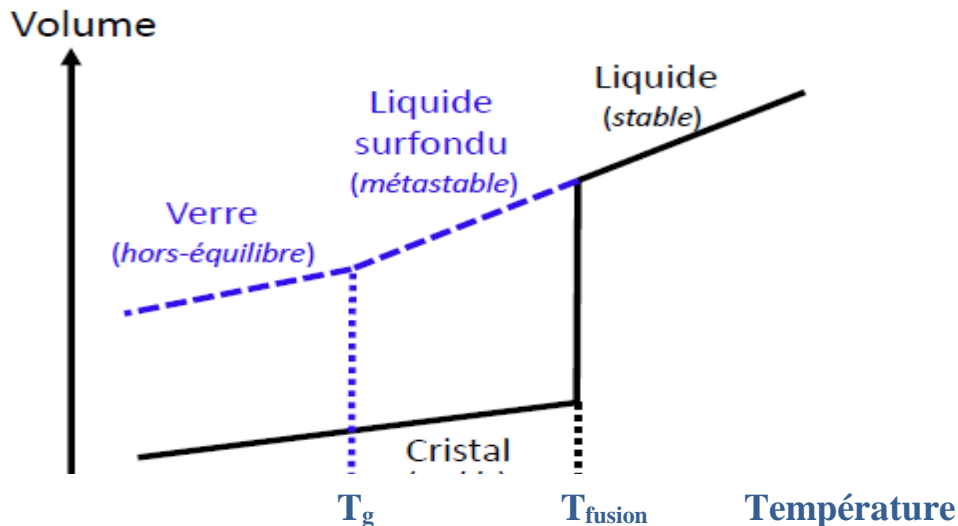


Figure 1.2 Different transition temperatures [1].

5-2 Cristaux liquides :

It's a complex fluid that combines the properties of a liquid phase and a crystallized solid phase. Its state is known as a mesophase or mesomorphic state (from the Greek for "of intermediate form"). The nature of the mesophase differs according to the nature and structure of the mesogen, the molecule at the origin of the mesophase, as well as temperature, pressure and concentration conditions. Liquid crystals are used in a number of applications, such as liquid crystal displays (LCDs), which use the properties of nematics, i.e. the molecules are distributed without positional order (as in a liquid), but remain on average parallel to one another. The property of changing color with respect to the angle of observation has also been used in banknotes, on certain logos, or certain decorative paints [2].

5-3 Granular states :

Depending on the energy supplied to them, granular materials can behave like solids, fluids or gases. This type of material is found in many fields, both natural (sand and dunes, rocks in a mudslide, avalanches, etc.) and industrial (preparation of medicines, manufacture of concrete, paints, granules, aggregates, etc.) or even agri-food (corn kernels, wheat kernels, etc.)[3].

5-4 Polymer :

A polymer is a macromolecule formed by the covalent linking of a very large number of repeating units derived from one or more monomers (which are also called units) and prepared from molecules called monomers.

The best-known polymers are :

- natural fibers :
- sugar polymers: slow sugars: starch, glycogen, and other fibers...
- nucleic acids: DNA and RNA are polymers of nucleosides linked by phosphate bridges;
- protein fibers: leather (collagen), silk and wool (keratin), etc. ;
- proteins, which are polymers of amino acids;
- plastics ;

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- natural (latex) and artificial rubbers;
- glues ;
- paints;
- resins.

They are widely used as matrixes for composite materials.

They have the following general properties [4] :

- They are "light" materials, with a low density (generally less than $1,500 \text{ kg}\cdot\text{m}^{-3}$, i.e. a density in relation to water of less than 1.5);
- They are generally flexible: thermoplastic polymers have a Young's modulus of less than about 3 GPa;
- They are stable only at moderate temperatures.

References :

- [1] http://www-ext.imPMC.upmc.fr/~ferlat/Enseignement/LP350/verres-LP350_2017.pdf
- [2] https://fr.wikipedia.org/wiki/Cristal_liquide
- [3] https://fr.wikipedia.org/wiki/Matériau_granulaire
- [4] <https://fr.wikipedia.org/wiki/polymère>

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Exercices :

Exercice 1 :

- 1- What is the volume of 4 moles of perfect gas if $P=3$ atm and $T=300\text{K}$?
- 2- A gas occupies a volume of 6 m^3 at a pressure of 1 atm. What happens to the pressure if the volume becomes equal to 4.5m^3 , while the temperature remains constant?
- 3- If the temperature of a gas increases from 0°C to 100°C at constant pressure, by how much will the volume change?

We give: Boltzmann's constant $k_B= 1.38 \cdot 10^{-23}$ J/K ;

the perfect gas constant $R=0.082$ l. atm. $\text{mol}^{-1} \cdot \text{K}^{-1}=8.31$ J. $\text{mol}^{-1} \cdot \text{K}^{-1}$

Exercice 2:

If we consider dry air as a homogeneous mixture of perfect gases, with molar mass

$M=29\text{ g}\cdot\text{mol}^{-1}$, made up of 78% nitrogen (N_2) and 21% oxygen (O_2), plus other gases present in small quantities. Air pressure at sea level is 1 atm. Calculate at 0°C :

- 1- The partial pressures of oxygen and nitrogen at sea level
- 2- The partial pressures of these two gases at an altitude of $h=6000$ m (the proportions of the gases are considered unchanged, and the molar mass of the mixture and the temperature are assumed constant).

We give $g=9.8\text{ m}\cdot\text{s}^{-2}$

Exercice 3:

At $T=300\text{K}$, two gases assumed to be perfect have root-mean-square velocities of 1367.7 m/s and 412.38 m/s respectively.

- 1- What are these two gases?
- 2- Calculate the mean translational kinetic energy in eV of the molecules of these gases at temperature

Personal work

Exercice1 :

The pressure p in a fluid varies according to the following law: $dp = -\rho g dz$, where ρ is the density, g is the acceleration of gravity and z is the altitude.

- 1- If the fluid is a liquid, of constant density ρ .
 - a- Find the expression for the variation in pressure as a function of altitude (hydrostatic law).
hydrostatics).

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b- Application: calculate the pressure at depth $h=5\text{m}$ of seawater if the pressure at its surface is 1atm . We give the density of seawater $\rho \approx 1.025 \text{ g/ml}$.

2- In the case where the fluid is a perfect gas, the density ρ is not constant.

a- Using the perfect gas law, find the expression for ρ as a function of pressure p , temperature T , perfect gas constant R and molar mass M .

b- Find the expression for the variation in pressure as a function of altitude z .

c- Application: calculate the pressure at altitude $z = 8600\text{m}$. The molar mass of dry air $M = 29 \text{ g/mol}$ is given.

Exercise2 :

Three containers respectively contain hydrogen, oxygen and nitrogen at temperature $T_1=293^\circ\text{K}$ under the following conditions: $\text{H}_2 : V_1=2.25 \text{ litres}$, $P_1=250 \text{ mm Hg}$, $\text{O}_2 : V_2=5.50 \text{ litres}$, $P_2=250 \text{ mm Hg}$, $\text{N}_2 : V_3=1,4 \text{ litres}$, $P_3=760 \text{ mm Hg}$

1- Calculate the mass m_1, m_2, m_3 of each gas, assuming they are perfect.

($1 \text{ atm} = 760 \text{ mmHg} = 1.013 \cdot 10^5 \text{ pascals}$)

2- Mix the three gases at the same temperature in a single container of volume $V_0=18.5 \text{ litres}$. The mixture thus formed is assumed to be ideal. Calculate the total pressure, the mole fraction of each gas and the various partial pressures. $M_{\text{H}}=1 \text{ g/mol}$, $M_{\text{O}}=16 \text{ g/mol}$, $M_{\text{N}}=14 \text{ g/mol}$ are given.

Exercise 3:

A tube of length $L=1 \text{ m}$ and cross-section $S=80 \text{ mm}^2$ contains N_{eon} ($M_{\text{Ne}}=20.2 \text{ g/mol}$) under a pressure of 1 KPa , at temperature $T=300\text{K}$.

1- Calculate the mass of Neon contained in the tube.

2- Calculate the internal energy and squared velocity of the gas molecules.

3- Add 0.4 mg of helium ($M_{\text{He}}=4\text{g/mol}$) to the tube;

a- What are the partial pressure of this gas and the mean square velocity of its molecules?

b- Calculate the total pressure and the total internal energy.

Answers :

Exercise 1 :

$$\begin{aligned} 1- \quad n_{gp} &= 4 \text{ moles} ; \quad R = 0.082 \text{ l.am.mol}^{-1} . \text{K}^{-1} \\ p &= 3 \text{ atm} \\ T &= 300 \text{ K} \end{aligned}$$

$$GP \rightarrow PV = nRT \Rightarrow V = \frac{nRT}{P} = \frac{4 \cdot 0,082 \cdot 300}{2} = 49,2 \text{ l}$$

$$\begin{aligned} 2- \quad V &= 6 \text{ m}^3 ; \quad V' = 4,5 \text{ m}^3 , \quad T = \text{cste} \\ P &= 1 \text{ atm} = 10^5 \text{ pas} \quad P' = ? ; \end{aligned}$$

$$PV = nRT \rightarrow n = \frac{PV}{RT}$$

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$$p'V' = n'RT \rightarrow n' = \frac{p'V'}{RT} \quad n' = n \quad \text{so } PV = P'V' \Rightarrow P' = \frac{PV}{V'}$$

$$p' = \frac{10^5 \cdot 6}{4,5} = 1,33 \cdot 10^5 = 1,33 \cdot 10^5 \text{ la} = 1,33 \text{ atm}$$

$$3- \quad T = 0^\circ\text{C} + 273; \quad P = \text{cste}$$

$$T' = 100^\circ\text{C} + 273$$

$$PV = nRT \quad \Leftrightarrow \frac{nRT}{V} = \frac{nRT'}{V'} \Leftrightarrow V' = V \frac{T'}{T}$$

$$PV' = nRT'$$

$$V' = V \frac{373}{273} = \left(1 + \frac{100}{273}\right)V = \left(1 + \frac{100}{273}\right)V = 1,36 V$$

$$\Delta V = V' - V = 1,36 V - V = 0,36 V$$

Exercice 2 :

$$M = 29 \frac{\text{g}}{\text{mol}}$$

$$78\% \text{ de } N_2 \rightarrow f_{N_2} = \frac{n_{N_2}}{nT} = 0,78$$

$$21\% \text{ de } O_2 \rightarrow f_{O_2} = \frac{n_{O_2}}{nT} = 0,21$$

$$P_{\text{air}} = 1 \text{ atm}$$

$$1- \quad P_{N_2} = f_{N_2} \cdot P_{\text{air}} = 0,78 \cdot 1 = 0,78 \text{ atm}$$

$$P_{O_2} = f_{O_2} \cdot P_{\text{air}} = 0,21 \cdot 1 = 0,21 \text{ atm}$$

$$2- \quad h = 6000 \text{ m}$$

$$P_{N_2} = f_{N_2} \cdot p_h$$

$$p_{O_2} = f_{O_2} \cdot p_h$$

p_h calculation? when $dz \uparrow \Rightarrow p \downarrow$

$dp = -\rho g dz$ (-) indicates the diminution of p When $dz \uparrow$

$$\rho = \frac{m}{V} \text{ air density } (m = nM)$$

$$dp = -\frac{m}{V} g dz = -n \frac{M}{V} g dz$$

$$\text{air considered as } \rho \text{ az perfect } \Rightarrow PV = nRT \Rightarrow \frac{n}{V} = \frac{p}{RT}$$

$$dp = -\frac{pM}{RT} g dz \Rightarrow \int_{p_o}^{p_h} \frac{dp}{p} = -\frac{Mg}{RT} \int_o^h dz$$

$$\ln \frac{p_h}{p_o} = -\frac{Mg}{RT} h \Rightarrow p_h = p_o e^{-\frac{Mg}{RT} h}, \quad M = 29 \cdot 10^{-3} \text{ kg mol}^{-1}$$

$$p_h = 1 e^{-29 \cdot 10^{-3} \cdot 9,81 \frac{6000}{8,314 \cdot 273}} \Rightarrow p_h = 0,47 \text{ atm}$$

$$P_{N_2} = f_{N_2} \cdot p_h = 0,78 \cdot 0,47 = 0,366 \text{ atm}$$

$$P_{O_2} = f_{O_2} \cdot p_h = 0,21 \cdot 0,47 = 0,0987 \text{ atm}$$

Exercise 3 :

$$T=300K$$

$$V_{q1}=1367,7 \text{ m/s}$$

$$V_{q2}=412.38 \text{ m/s}$$

1- We know that $M = \frac{3RT}{V_{q1}^2}$

$$M_1 = \frac{3RT}{V_{q1}^2} = \frac{3 \cdot 8,31 \cdot 300}{(1367,7)^2} = 3.998 \cdot 10^{-3} \text{ kg/mol} \approx 4\text{g/mol} , \text{ It's the He}$$

$$M_2 = \frac{3RT}{V_{q2}^2} = \frac{3 \cdot 8,31 \cdot 300}{(412.38)^2} = 43.97 \cdot 10^{-3} \text{ kg/mol} \approx 44\text{g/mol} , \text{ It's the CO}_2$$

$$\langle E \rangle = \frac{3}{2} k T = \frac{3}{2} \cdot 1.38 \cdot 10^{-23} \cdot 300 = 6.10^{-21} \text{ Joules}$$

$$1\text{eV}=1.6 \cdot 10^{-19} \text{ Joules}, \quad \langle E \rangle = 38.8 \text{ meV}$$

Chapter 2: General information on aqueous solutions

1- Study of solutions :

A solution is defined as any homogeneous mixture in a condensed phase (liquid or solid), i.e. a single phase comprising :
the solvent and the solute (or dissolved body). If one of the constituents is liquid, it is the solvent.

For example: in an aqueous solution of copper (solid) and ammonia (gas), the solvent is water; if two or more components are liquid, the most abundant liquid is the solvent. A distinction is made between neutral solutions, in which all solutes are molecules (molecules, neutral dissolved body particles), and electrolytic solutions, which are solutions of salts, bases (OH^-) and acids (H^+ , H_3O^+).

Electrolytic solutions conduct electric current

$\text{NaCl} \longrightarrow \text{Na}^+ + \text{Cl}^-$ strong electrolytic solution

$\text{CH}_3\text{COOH} \longrightarrow \text{CH}_3\text{COO}^- + \text{H}^+$ weak electrolyte solution

1-1 Dissociation coefficient :

This is the ratio of the number of moles of solute molecules that have dissociated to the initial number of moles of molecules. It is a dimensionless number

$\alpha = 1$ for a strong electrolyte solution

$\alpha < 1$ for a weak electrolytic solution

2- Concentrations :

The solution is characterized by the following parameters:

a- Molar fraction :

Is equal to the ratio of the number of moles of solute to the total number of moles in the solute+solvent mixture.

$$f_1 = \frac{n_1}{n_1 + n_2} \quad , \quad f_2 = \frac{n_2}{n_1 + n_2}$$

n_1 : number of molecules of one component

n_2 : number of molecules of the other component.

We see that $f_1 + f_2 = 1$

If there are several components, the mole fraction of the i-th component is

$$:f_i = \frac{n_i}{n_1 + n_2 + \dots + n_i + \dots + n_p}$$

Exemple1 :

Glucose solution obtained by dissolving 2g of glucose in 1 liter of water. The molar mass of glucose ($M_g = 180 \text{ g/mol}$) and the molar mass of water ($M_{\text{water}} = 18 \text{ g/mol}$) are given.

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$$f_g = \frac{ng}{nt} = \frac{ng}{ng+ne} ; ng = \frac{m}{M} = \frac{2}{180} = 0.01 \text{ mole}$$

$$ne = \frac{me}{M} ; V = 1 \text{ litre so } me = 1\text{kg} = 1000\text{g} \text{ therefore}$$

$$ne = \frac{1000}{18} = 55.56 \text{ mole}$$

(Knowing that for an aqueous solution 1 liter weighs about 1kg)

$$f_g = \frac{ng}{ng+ne} = \frac{0.01}{55.56+0.01} = \frac{0.01}{55.57} = 1.79 \cdot 10^{-4}$$

$$f_e = \frac{ne}{ng+ne} = \frac{55.56}{55.57} = 0.9998$$

a- Weight concentration :

This is the ratio of the mass of solute m_s to the volume of solution V_{sol}

$$C^p = \frac{m \text{ soluté}}{V \text{ solvant}} \text{ [g/l , g/cm}^3\text{]}$$

Note: the following approximation is often used $V_{\text{solution}} = V_{\text{solvent}}$

This expression is widely used, but is awkward because V_{solution} is highly temperature-dependent. For this reason, the weight concentration is defined as the ratio of the mass of solute m_s to the mass of solution or the mass of solvent :

$$C^p = \frac{m_s}{m_s+m_o} ; C^p = \frac{m_s}{m_o} \frac{(masse \text{ solute})}{(mase \text{ du solvant})} \quad (\text{g/kg})$$

If the solvent is water at room temperature, then the 2 expressions are equivalent, since 1 liter weighs approximately 1kilogram.

Example2: 500 cm³ of glucose solution at 7g/l is mixed with 2 liters of water. What is the weight concentration of the solution?

$$\text{In } 500 \text{ cm}^3 = 0.5 \text{ litre nous avons } C^p = 7 = \frac{m \text{ soluté}}{0.5}$$

$$m_{\text{solute}} (\text{glucose}) = 7 \cdot 0.5 = 3.5\text{g}$$

$$V_{\text{final solution}} = V_{\text{glucose}} + V_{\text{water}} = 0.5 + 2 = 2.5 \text{ litre}$$

$$C^p_{\text{solution}} = \frac{m_s \text{ glucose}}{V \text{ solution finale}} = \frac{3.5}{2.5} = 1.4 \text{ g/l}$$

a- Molarity and the molar concentration :

C^M of a solute (molarity) is the ratio of the number of moles of a solute to the volume of the solution.

$$C^M_{\text{solute}} = \frac{n \text{ solute}}{V \text{ solution}} \text{ (mole /l or SI mole/m}^3\text{)}$$

V_{solution} depends on temperature, so we define a molar concentration by weight

b- Molality :

the ratio of the number of moles of a solute to the mass of the solvent

$$C^m = \frac{m_{\text{solute}}}{m_{\text{solvent}}} \text{ (mole/kg)}$$

Exemple3:

dissolve 4.5 g NaCl (58.5 g/mol) in 500 cm³ distilled water. calculate C^M et C^m

$$C^M = \frac{n}{V_{\text{sol}}} = \frac{\frac{m}{M}}{V_{\text{sol}}} = \frac{\frac{4.5}{58.5}}{0.5} = 0,154 \text{ mole/l}$$

$$C^m = 0.154 \text{ mole/kg}$$

b- Ionic concentration or ionarity [ions] or C^I_{ions} ;

$$[\text{ions}] = \alpha U_{\text{ions}} C^M \text{ [ion gramme/l or mole d'ion/l]}$$

α : dissociation coefficient

U_{ions} : is the number of ions provided by each molecule as it dissociates

Exemple4 :



c- equivalent concentration : C^{eq}

Pour un ion i de concentration ionique $[i]$ et de valence z_i on a :

$$C_i^{eq} = |z_i| [i] = |z_i| \alpha \nu_i \text{ unit (gram equivalent/l or mole equivalent/l)}$$

ν_{ions} : number of ions supplied by the molecule when dissociating

Electrovalence of the solute molecule

Example5:

Given that the molar concentration of $\text{CaCl}_2 = 2 \text{ mole/l}$ with a dissociation coefficient $\alpha = 1$. Calculate the equivalent concentration of positive and negative ions. What conclusions can be drawn?



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$$C_{ca^{++}}^{eq} = |z_{ca^{++}}| [Ca^{++}] = |z_{ca^{++}}| \alpha \nu_{ca^{++}} C^M = 2 \times 1 \times 1 \times 2 = 4 \text{ Eqg/l}$$

$$C_{cl^{-}}^{eq} = |z_{cl^{-}}| [Cl^{-}] = |z_{cl^{-}}| \alpha \nu_{cl^{-}} C^M = 1 \times 1 \times 2 \times 2 = 4 \text{ Eqg/l}$$

We note that the equivalent concentration of positive ions is always equal to the equivalent concentration of negative ions, so we have

$$C_{+}^{eq} = C_{-}^{eq}$$

The total equivalent concentration of the solution C_{totale}^{eq} will be the sum of the equivalent concentrations of positive and negative ions.

g- Osmolar concentration or osmolarity C^0 :

The ratio of the number of all undissociated molecules and ions in a solution to the volume of the solution.

$$C^0 = \frac{n_{osm}}{V} = \frac{n_{ion}}{V} + \frac{n_{nondiss}}{V} \text{ osmole/l}$$

Consider a weak electrolyte solution of dissociation coefficient α of molar concentration C^M in a volume of solvent V . We'll look up the relationship between osmolar concentration and molar concentration.

Each molecule of this solute dissociates into ν positive ions noted by ν_{+} positive ions and ν negative ions noted by ν_{-} negative ions. Knowing that $C^M = \frac{n_{initial}}{V}$

$$C^0 = \frac{n_{osmol}}{V}$$

$$C^0 = \frac{n_{ions} + n_{nondiss}}{V}$$

$$\begin{aligned} n_{ions} &= n_{cations}^{ions+} + n_{anions}^{ions-} = \nu_{+} \alpha n_{init} + \nu_{-} \alpha n_{init} \\ &= (\nu_{+} + \nu_{-}) n_{diss} = \nu n_{diss} = \alpha \nu n_{init} \end{aligned}$$

$$n_{nondiss} = n_{initial} - n_{diss} = n_{initial} - \alpha n_{initial} = (1 - \alpha) n_{initial}$$

$$C^0 = \frac{n_{ions} + n_{nondiss}}{V} = \frac{\nu \alpha n_{init} + (1 - \alpha) n_{initial}}{V}$$

$$= \frac{n_{init} (\alpha \nu + 1 - \alpha)}{V} = (1 + \alpha(\nu - 1)) C^M$$

$$C^0 = (1 + \alpha(\nu - 1)) C^M$$

Avec $i = 1 + \alpha(\nu - 1)$, i is called the van't Hoff ionization coefficient

$$C^0 = i C^M$$

h- Ionic strength of a solution :

It is a numerical value that characterizes the state of the solution, and is unitless. It is generally denoted by μ and given by the formula

$$\mu = \frac{1}{2} [\sum_i z_i^2 [i]]$$

with μ : ionic strength, $[i]$: ionarity of ion i gram/l or mole/l and z_i : valency of ion i .

Exemple6 :

1 mole of CaCl_2 is taken with 2 moles of NaCl in a volume of 1 liter. Calculate the ionic strength of the solution.

$$z_{\text{Ca}^{++}} = +2 \quad \text{et} \quad z_{\text{Cl}^-} = -1$$

$$[\text{Ca}^{++}] = \nu_{\text{Ca}^{++}} \alpha C^M = 1 \times 1 \times 1 = 1 \text{ ion g/l}$$

$$[\text{Na}^+] = \nu_{\text{Na}^+} \alpha C^M = 1 \times 1 \times 2 = 2 \text{ ion g/l}$$

$$[\text{Cl}^-]_{\text{NaCl}} = \nu_{\text{Cl}^-} \alpha C^M = 1 \times 1 \times 2 = 2 \text{ ion g/l}$$

$$[\text{Cl}^-]_{\text{CaCl}_2} = \nu_{\text{Cl}^-} \alpha C^M = 2 \times 1 \times 1 = 2 \text{ ion g/l}$$

$$\mu = \frac{1}{2} [z_{\text{Na}^+}^2 [\text{Na}^+] + z_{\text{Cl}^-}^2 [\text{Cl}^-] + z_{\text{Ca}^{++}}^2 [\text{Ca}^{++}] + z_{\text{Cl}^-}^2 [\text{Cl}^-]]$$

$$= \frac{1}{2} [(1)^2 \cdot 2 + (1)^2 \cdot 2 + (2)^2 \cdot 1 + (-1)^2 \cdot (2)]$$

$$= \frac{1}{2} [2 + 2 + 4 + 2] = 5$$

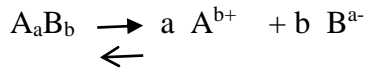
3- Solubility :

Solubility is the maximum amount of substance that can dissolve in a given volume of water. The solubility of a pure substance depends, for a given temperature, on the structure of the compound and the nature of the solvent.

To define the solubility product for a saturated solution (which is, by definition, a solution in dynamic equilibrium with undissolved solute), we define an equilibrium constant which is related to the solubility constant.

3-1 Equilibrium constant and solubility constant:

A weak solution is obtained by dissociation of a solute of the following form:



The equilibrium constant of this solution is given by Ostwald's law, which can be written as

$$:K = \frac{[A^{b+}]^a [B^{a-}]^b}{(A_a B_b)_{\text{nondiss}}}$$

The solubility constant K_s is defined by this equation:

$$K_s = [A^{b+}]^a [B^{a-}]^b \implies K_s = K (A_a B_b)_{\text{nondiss}}$$

Let C^M be the molar concentration of the electrolyte $A_a B_b$:

$$[A]^{b+} = a \alpha C^M \quad ; \quad [B]^{a-} = b \alpha C^M$$

$$\begin{aligned} (A_a B_b)_{\text{non diss}} &= \frac{n_{(A_a B_b) \text{ non diss}}}{V} = \frac{n_{\text{init}} - n_{\text{diss}}}{V} \\ &= \frac{n_{\text{int}} - \alpha n_{\text{int}}}{V} = \frac{(1-\alpha)n_{\text{int}}}{V} = (1-\alpha)C^M \end{aligned}$$

$$K = \frac{(a\alpha C^M)^a \cdot (b\alpha C^M)^b}{(1-\alpha)C^M}$$

Exemple1 :

$$1) \quad AB \begin{matrix} \rightarrow A^+ + B^- \\ \leftarrow \end{matrix} \quad K = \frac{\alpha C^M \alpha C^M}{(1-\alpha)C^M} = \frac{\alpha^2 C^M}{1-\alpha} \quad , \quad K_s = \alpha C^M \alpha C^M = \alpha^2 C^M$$

$$2) \quad AB_2 \begin{matrix} \rightarrow A^{+2} + 2B^- \\ \leftarrow \end{matrix} \quad K = \frac{(\alpha C^M)^1 (2\alpha C^M)^2}{(1-\alpha)C^M} = \frac{4 \alpha^3 C^M^3}{(1-\alpha)C^M} = \frac{4 \alpha^3 C^M^2}{(1-\alpha)} \quad , \quad K_s = 4 \alpha^3 C^M^3$$

4- Electrical properties of electrolytes :

4-1 Electrical conductivity:

Aqueous ionic solutions conduct electric current (conductors) because the coulombic force of attraction decreases, so the distance between the ions increases, the bond becomes weak and we have complete or partial dissolution of the crystal. On the other hand, liquids such as pure water, oil and glucose or urea solutions allow little or no electrical current to pass through them, as they are neutral solutions. The electrical resistance R of an electrolytic solution of resistivity ρ placed in a tank of length l and cross-section S has the expression :

$$R = \frac{\rho l}{S}$$

R is in Ohms, ρ in Ohms m, l : the length of the tank in m and S in m^2 .

Electrical conductivity χ is expressed in $\text{Ohms}^{-1} \text{m}^{-1}$, and is the inverse of resistivity; hence the expression for conductivity

$$\chi = \frac{1}{\rho}$$

Physical meaning of electrical conductivity χ :

An electrolytic solution is placed in a cylindrical cylindrical tank, across which a voltage

a voltage V (or ddp) is applied.

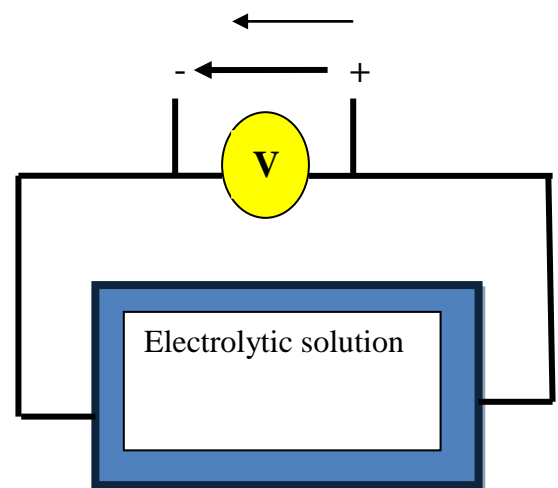
An electric current flows through

the tank, given by $i = \frac{V}{R}$

where R is the electrical resistance of the electrolyte solution.

$$i = \frac{V}{R} = \frac{VS}{\rho l} = \frac{V \chi S}{l}$$

$$\chi = \frac{i l}{S V} \implies \chi = \frac{i}{\frac{S}{V}}$$



$\frac{i}{S}$ Current density: current intensity per unit area

$\frac{V}{l}$ Electric field: applied voltage per unit length

Definition: the conductivity of an electrolytic solution is equal to the current density that flows through it when it is subjected to a unit electric field. ($E=1 \text{ V m}^{-1}$).

4.2 Ion mobility:

By definition, the ionic mobility μ of an ion is its velocity in a unit electric field. ($E = 1 \text{ V m}^{-1}$)

$$\vec{V} = \mu \vec{E}$$

An ion of charge q placed in a field E is subjected to a force $\vec{F} = q \vec{E}$.

Under the influence of force, the ion moves through the solvent, but is slowed down by the viscosity of the medium, expressed by the force f defined by Stokes' law:

$$\vec{f} = 6\pi\eta r \vec{v}$$

When the two forces F and f are equal, the ion moves at constant speed, hence

$$\sum \vec{F} = \vec{0} \quad q \vec{E} = 6\pi\eta r \vec{v}$$

$$\mu = \frac{q}{6\pi\eta r} = \frac{q}{f}$$

Avec $\vec{f} = 6\pi\eta r$

μ :ion mobility $\text{m}^2 \text{ V}^{-1} \text{ S}^{-1}$

r : ion radius

η : coefficient of viscosity of the medium, expressed in poiseuille ($\text{kg m}^{-1} \text{ s}^{-1}$) in the S.I. and in the c.g.s. system in poise ($\text{g cm}^{-1} \text{ s}^{-1}$)

4-3 Conductivity for an electrolytic solution:

Consider an electrolytic cell of cross-section S and length l subjected to a d d p V . We want to find the electric current intensity due to the displacement of ions under the effect of the electric field and ionic concentration. This is done for cations.

$$\mathbf{I} = \frac{dQ}{dt} \quad Q : \text{is the total charge carried by the cations}$$

$$Q_+ = z_+ e N_+ = z_+ e N n \quad \text{avec } (N = N_A n)$$

N_+ : number of cations

N : Avogadro number

$$[+] \text{ cation concentration given by this equation } \quad [+] = \frac{n_+}{V}$$

$$\text{so we have } Q_+ = z_+ e N [+] V$$

$$\text{In our volume element } dv \text{ we can write } dQ_+ = z_+ e N [+] dV$$

$$dQ_+ = z_+ e N [+] S dx$$

$$1 \text{ faraday} = 96500 \text{ coulombs mol}^{-1} = N e^- ; \quad (e^- = \text{electron charge})$$

$$F = 96500 \text{ C mol}^{-1}$$

Therefore $dQ_+ = z_+ F [+] S dx$ and the electric current is written as:

$$\mathbf{I}_+ = \frac{dQ_+}{dt} = z_+ F [+] S \frac{dx}{dt} = z_+ F [+] S \mathbf{v}_+$$

$$\mathbf{v}_+ = \mu_+ \mathbf{E}$$

$$\mathbf{I}_+ = z_+ F [+] S \mu_+ \mathbf{E}$$

Current intensity carried by the anions

The total current flowing through the tank $\mathbf{I} = \mathbf{I}_+ - \mathbf{I}_-$ donc

$$\mathbf{I} = \left(\sum_i |z_i| [i] |\mu_i| \right) F S \mathbf{E}$$

Current density \mathbf{J} is the ratio of electric current \mathbf{I} to cross-sectional area.

$$\mathbf{J} = \frac{\mathbf{I}}{\mathbf{S}} \quad \Longrightarrow \quad \mathbf{J} = \left(\sum_{\mathbf{i}} |\mathbf{z}_{\mathbf{i}}| [\mathbf{i}] |\boldsymbol{\mu}_{\mathbf{i}}| \right) \mathbf{F} \mathbf{E}$$

Conductivity of an electrolytic solution χ is the ratio of electric density to electric field.

$$\chi = \frac{\mathbf{J}}{\mathbf{E}} \quad \Longrightarrow \quad \chi = \left(\sum_{\mathbf{i}} |\mathbf{z}_{\mathbf{i}}| [\mathbf{i}] |\boldsymbol{\mu}_{\mathbf{i}}| \right) \mathbf{F}$$

Special case:

Consider a weak electrolyte solution consisting of ν_+ positive ions and ν_- negative ions.

$$[+] = \nu_+ \alpha C^M \quad ; \quad [-] = \nu_- \alpha C^M \quad \text{avec} \quad \nu_+ |\mathbf{z}_+| = \nu_- |\mathbf{z}_-| = \mathbf{Z}$$

\mathbf{Z} is the valence of the electrolyte, so for an electrolyte of valence \mathbf{Z} and molar concentration C^M

$$\chi = \left(|\mathbf{z}_+| [+] |\boldsymbol{\mu}_+| + |\mathbf{z}_-| [-] |\boldsymbol{\mu}_-| \right) \mathbf{F}$$

$$\chi = \mathbf{Z} \alpha C^M \left(|\boldsymbol{\mu}_+| + |\boldsymbol{\mu}_-| \right) \mathbf{F}$$

Equivalent cation conductivity is defined as

$$\Lambda_+ = \alpha |\boldsymbol{\mu}_+| \mathbf{F}$$

and anions

$$\text{And the equivalent solution conductivity} \quad \Lambda = \Lambda_+ + \Lambda_-$$

$$\chi = \mathbf{Z} C^M \Lambda \quad \text{Donc} \quad \Lambda = \frac{\chi}{\mathbf{Z} C^M}$$

Λ : Expressed in $\text{ohm}^{-1} \text{ m}^2 \text{ Eqg}^{-1}$

Exercices :

Exercice 1 :

An aqueous solution is obtained by dissolving 0.18g of glucose in 0.5 liters of water.

1-Calculate the molarity, molality and mass percentage (ratio of the mass of the solute to that of the solution X 100) of this solution, as well as the molar fractions of its various compounds, knowing that the molar mass of glucose is 180g/mol and that of water is 18g/mol.

2-Calculate the new molarity if 1 liter of water is added to the previous solution (dilution)?

3-Personal work: Recalculate the remaining quantities from question 1 for this same dilution.

Exercice 2:

Take 1 liter of a strong 0.3M CaCl₂ solution.

1-Calculate its ionic strength.

2-Personal work: Mix this solution with 2 liters of a 0.2M CaCl₂ solution. What is the ionic strength of the resulting solution?

Exercice 3:

Dissolve 12g of CH₃COOH in 1 liter of water. Calculate the concentrations of the various ions present in the solution, its equilibrium constant and its equivalent concentration, assuming a weak solution with a dissociation coefficient $\alpha=0.3$.

Exercice 4:

Dissolve 39.63g (NH₄)₂SO₄ (M=132.1g/mol) in 1 litre of water. Assuming partial solute dissociation of dissociation coefficient =0.8 .

1-Calculate the osmolarity of the resulting solution and its freezing temperature (K_c=1,86 °C.Osm⁻¹.ℓ).

2-Personal work :

-Calculate the equilibrium constant of this solution.

-What would be the freezing temperature of a urea solution of the same molarity as the previous solution?

Exercice 5:

Consider an electrolytic cell of length $l= 25$ cm, cross-section $S= 15\text{cm}^2$ and resistance $R=45\Omega$.

Calculate the constant of this cell, its conductivity and the intensity of the electric current flowing through it, given that it is subjected to a potential difference $U=60\text{mV}$.

Exercice 6:

Consider a strong 0.05M NaCl solution, subjected to an electric field $E=9.10^{-6}$ V/m.

Knowing that the mobility of Na⁺ ions is equal to $5.1.10^{-8}$ m² V⁻¹s⁻¹ and that that of Cl⁻ ions is equal to $-7.9.10^{-8}$ m² V⁻¹s⁻¹, calculate the limiting velocities of these two ions, their

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coefficients of friction as well as the equivalent conductivity and the conductivity of the solution. We give $F=96500 \text{ C}\cdot\text{mol}^{-1}$.

Answers :

Exercise 1 :

$$mg=0.18 \text{ g} \quad , \quad Mg=180\text{g/mol}$$

$$V=0,5 \text{ l}$$

$$1- \text{ the molarity } C^M = \frac{n}{V} = \frac{m}{MV} = \frac{0,18}{180 \cdot 0,5} = 0,002 \text{ mole/l}$$

$$\text{the molarity } C^m = \frac{n_{\text{solute}}}{m_{\text{solution}}} = \frac{m}{V m_{\text{solution}}} = \frac{0,18}{180 \cdot 0,05}$$

$$C^m = 0,002 \text{ mole/kg}$$

$$C^m = 0,2\%$$

$$ng = \frac{mg}{M} = \frac{0,18}{180} = 0,001 \text{ mole}$$

$$neau = \frac{neau}{M} = \frac{0,5 \cdot 1000}{18} = 27,77 \text{ mole}$$

Molar fractions of water and glucose

$$fg = \frac{ng}{ng + neau} = \frac{0,01}{0,01 + 27,77} = \frac{0,01}{27,787} = 0,000359$$

$$feau = \frac{neau}{ng + neau} = \frac{27,77}{0,01 + 27,77} = \frac{27,77}{27,787} = 0,999$$

2- If 1 L of water is added to the previous solution, the new volume is

$$V'=0,5+1=1,5\text{l}$$

$$C^m = \frac{n}{V'} = \frac{0,18}{180 \cdot 1,5} = 0,0006 \text{ mole/l}$$

Exercise 2 :



$$V=1 \text{ litre}$$

$$C^M=0,3 \text{ mole/l}$$

$$F = \frac{1}{2} \left[\sum_i |z_i|^2 [i] \right]$$

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$$= \frac{1}{2} \left[|zCa^{++}|^2 [Ca^{++}] + |zcl^-|^2 [cl^-] \right]$$

$$[Ca^{++}] = \alpha \cdot \vartheta C^M = 1 \cdot 1 \cdot 0,3 = 0,3 \text{ mole of ion/l}$$

$$[cl^-] = 2 \cdot 1 \cdot 0,3 = 0,6 \text{ mole of ion/l}$$

$$F = \frac{1}{2} [(2)^2 \cdot 0,3 + (-1)^2 \cdot 0,6]$$

$$F = \frac{1}{2} [1,2 + 0,6] = 0,9$$

Personal work

$$F = \frac{1}{2} \left[\sum_i |z_i|^2 [i] \right]$$

$$\frac{1}{2} = [|zCa^{++}|^2 [Ca^{++}] + |3cl^-|^2 [Cl^-] + |3Ca^{++}|^2 [Ca^{++}] + |3Cl^-|^2 [Cl^-]]$$

First solution

$$V' = 2 + 1 = 3l$$

$$[Cl^-]_1 = \alpha \cdot \vartheta \cdot C_1^M = 1 \cdot 2 \cdot 0,3 \cdot \frac{1}{3} = 0,2 \text{ mole of ion/l}$$

$$[Ca^{++}]_2 = \alpha \cdot \vartheta \cdot C_2^M = 1 \cdot 1 \cdot 0,2 \cdot \frac{2}{3} = 0,1333 \text{ mole of ion/l}$$

$$[Cl^-]_2 = 0,266 \text{ mole of ion/l}$$

$$F = \frac{1}{2} [4 \cdot 0,1 + 1 \cdot 0,2 + 4 \cdot 0,133 + 1 \cdot 0,266]$$

$$F = 1,3992$$

Exercice 3:



$$[cH_3Coo^-] = \alpha \cdot \vartheta \cdot C^M = 0,3 \cdot 1 \cdot 0,2 = 0,06 \text{ mole of ion/l}$$

$$C^M = \frac{n}{V} = \frac{m}{MV} = \frac{12}{60 \cdot 1} = 0,2 \text{ mol/l} \quad [H^+]$$

$$M = 12 \cdot 2 + 4 + 16 \cdot 2 = 60 \text{ g/mol}$$

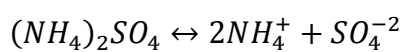
$$k = [cH_3Coo^-] \frac{[H^+]^1}{(1-\alpha)C^M} = (0,06)^1 \frac{(0,06)^1}{(1-0,3) \cdot 0,2} = 0,0257$$

$$C_{cH_3Coo^-}^{eq} = |z_{cH_3Coo^-}| [cH_3Coo^-] = 1 \cdot 0,06 \text{ eq g/l}$$

$$C_{H^+}^{eq} = 0,06 \text{ Eqg/l}$$

$$C_{solution}^{eq} = 0,06 \cdot 2 = 0,12 \text{ Eq g/l}$$

Exercice 4 :



$$C^0 = iC^M$$

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$$C^M = \frac{n}{V} = \frac{m}{MV} = \frac{39,63}{132,11} = 0,3 \text{ mol/l}$$

$$i = (1 + \alpha (\vartheta - 1)) = 1 + 0,8(3 - 1) = 2,6$$

$$C^o = 2,6 \cdot 0,3 = 0,78 \text{ osmole/l}$$

$$\Delta\theta = K_c C^o = 1,86 \cdot 0,78 = 1,4508 C^o$$

Personal work:

$$K = \frac{[NH_4^+]^2 [SO_4^{2-}]^1}{(1 - \alpha) C^M} = (0,48)^2 \frac{(0,24)^1}{(1 - 0,8)0,3}$$

$$K = 0,9216$$

$$[NH_4^+] = \alpha \cdot \vartheta \cdot C^M = 0,8 \cdot 2 \cdot 0,3 = 0,48 \text{ mole of ion/l}$$

$$[SO_4^{2-}] = 0,81 \cdot 1 \cdot 0,3 = 0,24 \text{ mole of ion/l}$$

$$\Delta\theta = K_c C^o = 1,86 \cdot 0,3 = 0,558 C^o$$

$$C^o = i C^M = 0,3 \text{ mol/l}$$

$$i = 1 + \alpha (\vartheta - 1) = 1$$

$$\alpha = 0 \text{ for urea}$$

Exercice 5 :

$$R = \rho \frac{L}{S}$$

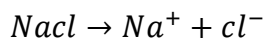
$$\text{The tank constant } L/S = 25/15 = 1,66 \text{ m}^{-1}$$

$$\rho = R \frac{S}{L} \quad \text{with } \rho = \frac{1}{\chi}$$

$$R = \frac{L}{\chi S} \rightarrow \chi = \frac{L}{R S} = \frac{25 \cdot 10^{-2}}{45 \cdot 15 \cdot 10^{-4}} = 3,7 \quad \Omega^{-1} \text{m}^{-1}$$

$$V = R i \rightarrow i = \frac{V}{R} = 60 \frac{10^{-3}}{3,70} = 0,00162 \text{ A}$$

Exercice 6 :



$$E = 9 \cdot 10^{-6}$$

$$C^M = 0,05 \text{ mole/l}$$

$$v = \mu E$$

$$v_+ = |\mu_+| E = 5,110^{-8} \cdot 9 \cdot 10^{-6} = 4,59 \cdot 10^{-13} \text{ m/s}$$

$$v_- = |\mu_-| E = 7,9 \cdot 10^{-8} \cdot 9 \cdot 10^{-6} = 7,11 \cdot 10^{-13} \text{ m/s}$$

$$\mu = \frac{q}{f} \rightarrow f = \frac{q}{\mu}$$

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$$f_+ = \frac{q}{|\mu_+|} = 1,6 \frac{10^{-19}}{5,1 \cdot 10^{-8}} = 3,13 \cdot 10^{-12} \text{ kg/s}$$

$$f_- = \frac{q}{|\mu_-|} = 1,6 \frac{10^{-19}}{7,9 \cdot 10^{-8}} = 2,02 \cdot 10^{-12} \text{ kg/s}$$

$$\tau_+ = \alpha |\mu_+| F = 1,5 \cdot 10^{-8} \cdot 96500 = 0,00492 \text{ } \Omega^{-1} (\text{eqg})^{-1} \text{m}^2$$

$$\tau_- = \alpha |\mu_-| F = 1,7 \cdot 10^{-8} \cdot 96500 = 0,00762 \text{ } \Omega^{-1} (\text{eqg})^{-1} \text{m}^2$$

Additional exercises

Exercise 1 :

Consider 0.5 liters of water ($M_{\text{water}}=18 \text{ g} \cdot \text{mol}^{-1}$) containing 24g urea ($M_u=60 \text{ g} \cdot \text{mol}^{-1}$) and 36 g glucose ($M_g=180 \text{ g} \cdot \text{mol}^{-1}$). Calculate the mole fractions, molar concentrations and weight concentrations of this solution.

Exercise 2:

Consider a solution obtained by dissolving 4.25 g of fully dissociated barium chloride of formula BaCl_2 in a volume of $V=200 \text{ ml}$ of water.

Data: $M(\text{Cl}) = 35.5 \text{ g} \cdot \text{mol}^{-1}$ $M(\text{Ba}) = 137.3 \text{ g} \cdot \text{mol}^{-1}$

Calculate the molarity, molality, weight concentration and ionic concentrations of the cations and anions.

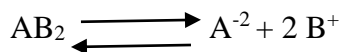
Exercise 3:

1-Aqueous solution of CaSO_4 with a molar concentration of $C_M=0.2 \text{ mol/l}$, totally dissociated into Ca^{++} and SO_4^{-2} . Calculate its osmolarity and ionic strength.

2- If we take 1 liter of this solution and add 2 liters of pure water, what is the ionic strength of the resulting solution?

Exercise 4:

An electrolyte AB_2 partially dissociates in water as follows



The cryoscopic lowering of this decimolar solution is 0.36°C .

1- What is the dissociation coefficient of this solution?

2- What is the equilibrium constant of this solution?

We give: Cryoscopic constant of water $K_c=1.86^\circ\text{C} \cdot \text{osmol}^{-1} \cdot \text{l}$

Answers :

Exercise 1 :

Calculate the molar fractions, molar concentrations and weight concentrations of this solution

Molar fractions :

Molar fractions of urea

$$n_{urea} = \frac{m_{urea}}{M_{urea}} = \frac{24}{60} = 0,4 \text{ mole}$$

$$n_g = \frac{mg}{Mg} = \frac{36}{180} = 0,20 \text{ mole}$$

$$n_{eau} = \frac{m_{water}}{M_{water}} = \frac{0,5 \cdot 1000}{18} = 27,77 \text{ mole}$$

$$\text{for water : } 1\text{l} \rightarrow 1 \text{ kg} \Rightarrow n_T = n_{urée} + n_g + n_{eau} = 0,4 + 0,20 + 27,77$$

$$n_T = 28,37 \text{ mole}$$

$$0,5\text{l} \rightarrow 0,5\text{kg} \rightarrow 0,5 \cdot 1000\text{g}$$

$$f_{urea} = \frac{0,4}{28,37} = 0,0140 ; C_{urea}^M = \frac{n_{urea}}{V} = \frac{0,4}{0,5} = 0,8 \text{ mole/l}$$

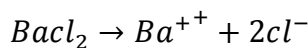
$$f_g = \frac{0,20}{28,37} = 0,00704 ; C_g^M = \frac{n_g}{V} = \frac{0,20}{0,5} = 0,4 \text{ mole/l}$$

$$f_{water} = \frac{27,77}{28,37} = 0,978 ; C_{water}^M = \frac{n_{water}}{V} = \frac{27,77}{0,5} = 55,54 \text{ mole/l}$$

$$C^P = C^M \cdot M$$

$$C_{urea}^P = 0,8 \cdot 60 = 48 \text{ g/l} ; C_g^P = 72 \text{ g/l} ; C_{eau}^P = 999,72 \cong 1000 \text{ g/l}$$

Exercise 2 :



$$M_{BaCl_2} = 137,3 + 35,5 \cdot 2 = 208,3 \text{ g/mol}$$

molarity C^M

$$C^M = \frac{n}{V} = \frac{m}{MV} = \frac{4,25}{208,3 \cdot 0,2} = 0,102 \text{ mole/l}$$

molality C^m

$$C^m = \frac{n_{solute}}{m_{solvent}} \text{ (mole/kg)}$$

$V_{water} = 0,2 \text{ l} \rightarrow 0,2 \text{ kg}$ because 1 liter weighs about 1kg for an aqueous solution

$$C^m = \frac{n_{\text{soluté}}}{m_{\text{solvant}}} = \frac{4,25}{208,3 \cdot 0,2} = 0,102 \text{ mole/kg}$$

ionic concentration

$$[i] = \alpha \cdot \nu \cdot C^M$$

$$[Cl^-] = \alpha \cdot \nu_{Cl^-} \cdot C^M = 1 \cdot 2 \cdot 0,102 = 0,204 \text{ mole of ion/l}$$

$$[Ba^{++}] = \alpha \cdot \nu_{Ba^{++}} \cdot C^M = 1 \cdot 1 \cdot 0,102 = 0,102 \text{ mole of ion/l}$$

Exercice 3 :



$$1- C^0 = i \cdot C^M$$

$$i = 1 + \alpha(\nu - 1) = 2, \quad C^0 = 2 \cdot 0,2 = 0,4 \text{ osmole/l}$$

$$F = \frac{1}{2} [Z_{Ca^{++}}^2 [Ca^{++}] + Z_{SO_4^{-2}}^2 [SO_4^{-2}]]$$

$$[Ca^{++}] = \alpha \cdot \nu_{Ca^{++}} \cdot C^M = 1 \cdot 1 \cdot 0,2 = 0,2 \text{ mole of ion/l}$$

$$[SO_4^{-2}] = \alpha \cdot \nu_{SO_4^{-2}} \cdot C^M = 0,2 \text{ mole of ion/l}$$

$$F = 0,5 (4 \cdot 0,2 + 4 \cdot 0,2) = 0,8$$

$$2- V' = 3 \text{ l}$$

$$C^{M'} = \frac{C_M^1 V_1}{V'} = \frac{0,2 \cdot 1}{3} = 0,066 \text{ mole/l}$$

$$[Ca^{++}] = \alpha \cdot \nu \cdot C^{M'} = 1 \cdot 1 \cdot 0,066 = 0,066 \text{ mole of ion/l}$$

$$[SO_4^{-2}] = 0,066 \text{ mole of ion/l}$$

$$F = 0,5 (4 \cdot 0,066 + 4 \cdot 0,066) = 0,264$$

Chapter III: Surface phenomena

1- Surface tension :

The experiment shows some simple observations that highlight these phenomena:

- 1- Droplet formation on a plant leaf
- 2- Ascent of liquids in capillary tubes
- 3- A solid body can float on the surface of water.
- 4- A meniscus exists on the surface of a liquid, in contact with the container wall.
- 5- The presence of soap bubbles

These experiments demonstrate the existence of a force f at the surface of a liquid. This is a consequence of the specific properties of the liquid surface, which are different from those of the rest of the liquid.

1-1. Definition :

Inside the liquid, each molecule is subject to electrostatic Van der Waals-type forces of attraction, which arise due to the polarity of certain molecules, with the resultant force being zero.

On the other hand, surface molecules are essentially attracted towards the interior of the liquid, as air molecules are too few in number to achieve equilibrium; the resultant of the attractive forces of surface molecules is therefore directed towards the interior of the liquid, tending to compress the liquid and thus reduce its free surface.

Consequently, there is a force, f , between the interface of a liquid and another medium. Surface tension σ is measured by the ratio of the force f to the length l over which it acts.

$$\sigma = \frac{f}{l}$$

σ en N.m^{-1} ; f en N ; l en m .

Surface tension depends on the liquid and the nature of the other part of the interface (gas, air, glass, metal...).

Ex: water / air,

$$\sigma = 73 \cdot 10^{-3} \text{ N.m}^{-1}. \text{ huile/air } \sigma = 32 \cdot 10^{-3} \text{ N.m}^{-1}.$$

1-2 Adhesion of a liquid to a solid :

This is the work required to separate a liquid from a solid over an area of 1 m^2 . We call σ_L the surface tension of the liquid; σ_s the surface tension of the solid and that of the liquid-solid interfacial tension σ_{SL} . By separating the liquid from the solid, over 1 m^2 , we also create 1 m^2 of liquid-air interface, 1 m^2 of solid-air interface; the solid-liquid adhesion energy is therefore

$$W_{SL} = (\sigma_s + \sigma_L - \sigma_{SL})S$$

In this equation, only σ_L is known. The difference ($\sigma_S - \sigma_{SL}$) can be measured relatively easily, whereas the two terms cannot be measured separately.

1-3 Contact angle θ :

Considering the equilibrium of a liquid drop on a horizontal solid surface. Figure 3.1 clearly shows the existence of three (03) phases.

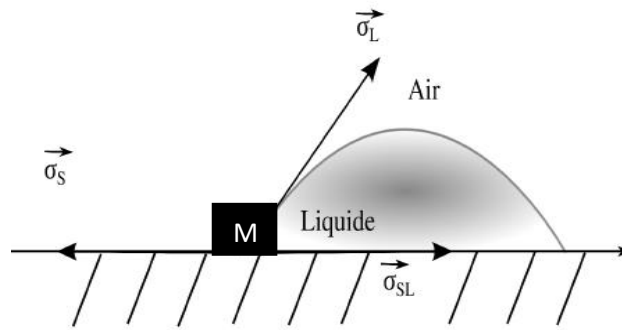


Figure 3.1

The equilibrium of point M is written (horizontal components of surface forces) :

$$\sigma_s = \sigma_{SL} + \sigma_L \cos(\theta)$$

In other words

$$\sigma_s - \sigma_{SL} = \sigma_L \cos(\theta) \implies \cos(\theta) = \frac{\sigma_s - \sigma_{SL}}{\sigma_L}$$

The angle θ is called the contact angle. It can be measured. The work of adhesion can be written as :

$$W_{SL} = \sigma_L (1 + \cos \theta) S .$$

We can see that the smaller the angle θ , the greater the work of adhesion. Consider the following cases:

- $\theta = 0^\circ$, $W_{SL} = 2 \sigma_L S = W_L$: the work of adhesion is equal to the work of cohesion of the liquid. This is the extreme case of equilibrium, corresponding to complete spreading of the liquid.

The condition $W_{SL} = W_L$ is the limiting case of perfect wetting. A priori, wetting is perfect if $W_{SL} \geq W_L$.

- $\theta = 180^\circ$, $W_{SL} = 0$: the work of adhesion is zero. Wetting is zero.

- $0 < \theta < 90^\circ$ (acute) to good but not perfect wetting.

- $90^\circ < \theta < 180^\circ$ to a bad but non-zero wetting.

1-4 Examples of dampening applications :

In phytopharmacy: certain wetting agents are used in pharmacy to promote the penetration of drugs through the skin or mucous membrane.) Conversely, anti-wetting agents can be sought to reduce the cutaneous penetration of toxic products.

In the case of anti-parasite products, the liquid must adhere perfectly to the surface to block the insect's pores, thus preventing it from breathing. So, with a contact angle $\theta=0$, we have a wetting product.

In the case of weedkillers, on the other hand, wetting must be poor, as these products must not be harmful to insects, so we must choose anti-wetting products. Poor wetting may be due to a layer having formed on the surface of the solid, so this surface must first be treated (it must be well polished).

To determine a liquid's adhesion, or wetting, it is therefore important to accurately determine its surface tension.

There are several methods for measuring surface tension, including the capillary rise method (Jurin's law), the immersed slide method and the droplet method (Stalagmometry). In the following, we will describe the capillary rise method in detail

1- Capillary action :

2-1 definition:

It is the phenomenon that explains, among other things, the rise (verticality) of liquids, against the effect of gravity, along a surface because of their surface tension.

2-2 Capillary Ascension Method(Law of Jurin) :

A tube made from a solid, whose wettability is defined by an angle θ , is immersed in a liquid. For good wetting, the liquid level rises $h>0$ Figure3.2(a). With poor wetting, the liquid level falls Figure3.2(b).

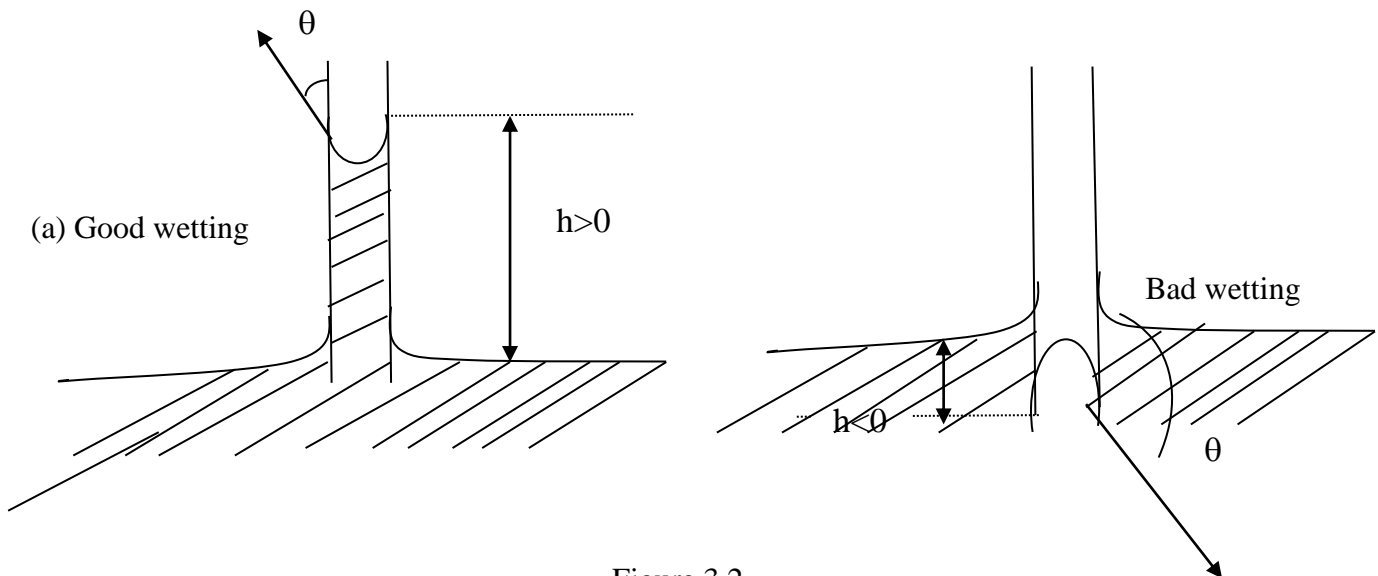


Figure 3.2

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The smaller the radius or meniscus, the greater the difference in level h.

Let ΔF be the attraction exerted by the tube on the inner liquid

$$\Delta F = 2 \pi r \sigma \cos\theta$$

Equilibrium will be established when the attraction and the weight of the column compensate each other.

The weight of the liquid in the column $P = h \pi r^2 \rho g$

g: intensity of gravity

h: height without meniscus

ρ : density of the liquid

$h \pi r^2$: volume of liquid column

Equating the two equations $\Delta F = P$ gives us:

$$\sigma \cos(\theta) = \frac{h r \rho g}{2} \quad \text{Jurin's law}$$

2-3 Surface tension measurement (capillary method)

By applying Jurin's law, we deduce a value for σ from the measurement of the height difference h and knowledge of the other parameters.

1- Surface tension force (liquid-gas interfaces) :

Consider a molecule (A) inside the liquid. It is surrounded on all sides by other molecules. If we neglect the influence of thermal agitation, the resultant of the forces of attraction experienced by molecule (A) is on average zero, since all directions are equivalent.

$$\sum \vec{F} \equiv \vec{0}$$

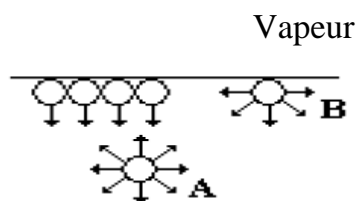


Figure 3.3

For another molecule (B) in Figure 3.3, located on the surface of the liquid, if we neglect the attraction of gaseous molecules, this molecule is drawn inwards on average perpendicular to the surface by neighboring liquid molecules. The same applies to all surface molecules, which thus form a kind of taut film that compresses the interior of the liquid. To bring the molecule (A) to the surface, i.e. to increase the surface area, the energy required to move a molecule from the liquid phase to the vapour phase must be supplied.

Any increase in surface area (ΔS) requires external energy (ΔE), such as $\Delta W = \Delta E = \sigma \Delta S$

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The coefficient of proportionality σ , which is always positive, is called surface tension. Its unit is N/m or J/m².

Another definition still in use is to represent σ as a force (F) tangent to the surface.

Some surface tension measurements

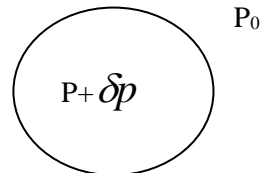
Liquides (20°C)	σ (dynes/cm)
H ₂ O	72,8
Glycerin	60
Glycol	46
Benzene	28.5

Molten metals have surface tensions of the order of 500 dynes/cm.

The liquid-gas surface tension phenomenon will create excess pressure inside a drop or column.

3-1 Overpressure inside a drop :

Consider a spherical drop in figure 3.4. The membrane surrounding the drop exerts an overpressure inside the drop. δp



P_0 : atmospheric pressure outside the drop
 $= 1.013 \cdot 10^5$ N/m² (pascals)

P : internal pressure.

To calculate δp , we need to increase the volume of the drop from radius r to radius $r + \Delta r$, so we need to supply external energy, ΔE

The work supplied to the system is $\Delta W = \frac{\delta F}{S} S \Delta x = \delta p \Delta V$

$$\Delta W = \delta p \Delta V = \delta p \Delta \left(\frac{4}{3} \pi r^3 \right) = \delta p 4\pi r^2 \Delta r \quad -1-$$

This work increases the surface area of the drop.:

$$\Delta W = \sigma \Delta S = \sigma \Delta (4\pi r^2) = \sigma 8\pi r \Delta r \quad -2-$$

Equalizing -1- and -2- we obtain :

$$p - p_0 = \delta p = \frac{2\sigma}{r} \quad \text{Laplace's law for a drop.}$$

Laplace's law is a relationship that links the pressure difference between the 2 faces of an elastic membrane or liquid film to the surface tension σ .

3-1 Overpressure inside a soap bubble :

Consider a soap bubble consisting of two membranes, an inner membrane of radius r and an outer membrane of radius $r + \delta r$ Figure 3.5

σ surface tension of the soap solution

r : radius of inner membrane

$r + \delta r$ radius of outer membrane

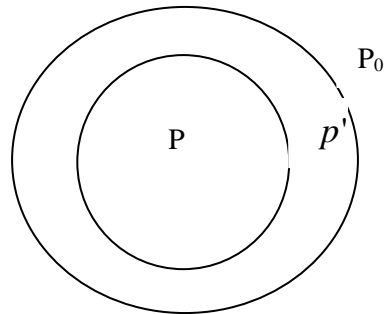


Figure 3.5

$$p - p' = \frac{2\sigma}{r} \quad \text{-1- overpressure created by the inner membrane}$$

$$p' - p_0 = \frac{2\sigma}{r + \delta r} \quad \text{-2- overpressure created by the outer membrane}$$

Adding these two equations gives us :

$$\delta p = p - p_0 = \frac{2\sigma}{r + \delta r} + \frac{2\sigma}{r} = \frac{4\sigma}{r} \quad \delta r \ll \ll \ll \ll \ll$$

δp Overpressure inside the bubble compared with outside. So the overpressure inside a soap bubble is 2 times greater than that inside a drop.

3-3 Biological application: respiration in living beings

The surface area of the lungs is increased by the presence of alveoli. Expansion of the lungs requires considerable work, as the surface tension that glues the alveolar membranes together is high. To facilitate ventilation, surfactants reduce the surface tension on the inner surface of the alveoli.

The presence of these surfactants reduces the work required to expand the lungs:

As the alveolus expands, the concentration of surfactants per unit area decreases, and surface tension increases. Resistance to expansion increases, protecting the alveoli from bursting [1].

P_0 : Liquid pressure

P_i : Pressure inside of the cell

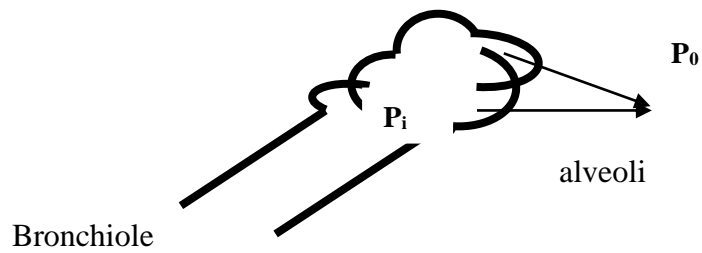


Figure 3.6 Respiratory function in living organisms

[1] Kane/Sternheim Physique InterEditions

2- Adsorption :

Adsorption is a surface phenomenon by which atoms, ions or molecules (adsorbates) attach themselves to a solid surface (adsorbent) from a gaseous phase, liquid or solid solution.

Exercices :

Exercice 1

A 5 cm square metal frame is placed in a bath of fuel oil. To separate the frame from the liquid, a force of $7.32 \cdot 10^{-3} \text{N}$ must be exerted. Calculate the surface tension of the fuel oil (remember that separating the frame from the liquid creates two surfaces).

Exercice 2

A liquid has a surface tension coefficient $\sigma = 25 \cdot 10^{-3} \text{ N.m}^{-1}$. Blow a soap bubble of radius $r = 3 \text{ cm}$ with this liquid.

1. Calculate the overpressure inside the bubble.
2. Calculate the total work expended to blow the bubble.

Exercice 3

1. A glass-wetting liquid of density $\rho = 1.05 \cdot 10^3 \text{ kg.m}^{-3}$ rises to an average height $h = 1.5 \text{ cm}$ in a vertical glass capillary tube of internal diameter $d = 1 \text{ mm}$. Calculate the surface tension coefficient of the liquid.

2. What is the height reached in the same capillary tube if it is immersed vertically in mercury? Given: $\sigma_{\text{Hg}} = 500 \cdot 10^{-3} \text{ N/m}$, $\rho_{\text{Hg}} = 13600 \text{ kg.m}^{-3}$,

$$\Theta = 135^\circ.$$

Exercice 4

Consider a tube of internal diameter d immersed vertically in a liquid of surface tension σ and density ρ . Assume perfect wettability and denote by h the difference in level of the liquid in the tube.

With water, $h_0 = 92.3 \text{ mm}$. We give: $\rho_0 = 0.9973 \cdot 10^3 \text{ kg.m}^{-3}$, $\sigma = 71.93 \cdot 10^{-3} \text{ N.m}^{-1}$. For benzene, $h = 42.4 \text{ mm}$. Deduce the surface tension coefficient of benzene, given that its density ρ is $0.8840 \cdot 10^3 \text{ kg.m}^{-3}$.

Additional exercises

Exercice 1

1. Calculate the overpressure in a drop of water with a radius of 1 mm ($\sigma = 73.6 \text{ mN/m}$).
2. Calculate the overpressure in a soap bubble of 1 cm radius ($\sigma = 30 \text{ mN/m}$).

Exercice 2

A metal ball of radius r and density $\rho_m = 7900 \text{ kg.m}^{-3}$ floats on water ($\sigma = 70 \text{ mN/m}$ and $\rho = 1000 \text{ kg.m}^{-3}$). What is the value of r if the liquid is in contact with the sphere along a meridian circle?

Ans : $r = 1.228 \text{ mm}$

Exercise 3

In plants, the ascent of sap is favored by the wetting of the inner walls of circulation ducts. The height of ascent h is obtained by applying Jurin's law.

1. Demonstrate Jurin's law of capillary rise.
2. Calculate the height h for a radius of the order of $10 \text{ }\mu\text{m}$, the contact angle θ being zero.

Given: $\sigma = 70 \text{ mN/m}$ and $\rho = 1000 \text{ kg}\cdot\text{m}^{-3}$.

Ans : $h \approx 1.4 \text{ m}$

Exercise 4

A capillary tube was calibrated at 20°C with water that rose 8.31 cm to reach equilibrium ($\Theta = 0^\circ$). Using the same capillary, a sample of mercury was lowered by 3.67 cm . = 13595 Kg/m^3

- a) Knowing that water $\sigma = 73 \cdot 10^{-3} \text{ N/m}$, evaluate for mercury ($\Theta = 180^\circ$),
- b) What is the diameter of the capillary used?

Exercise n°5

1. At the end of expiration, the total surface area of the pulmonary alveoli is 75 m^2 . Their volume is 3 l . Calculate the average radius of an alveolus and the total number of alveoli if we consider them to have a spherical shape.
2. During inspiration, lung volume increases by 1.5 l . Calculate the increase in alveolar surface area.
3. The alveolar surface is covered by a lipid film with a surface tension of 20 mN/m . Calculate the energy required to increase the surface area of the alveoli during inspiration.
4. Since the surface tension of the alveolar surface has become equal to 50 mN/m due to pathological circumstances, calculate the energy required for inspiration in this case.

Answer : 1. $r = 0.12 \text{ mm}$, $n = 4.14 \cdot 10^8 \text{ alvéoles}$; 2. $\Delta S = 23 \text{ m}^2$; 3. $W = 0.46 \text{ J}$; 4. 1.15 J .

Answers

Exercise 1 :

Pull-out force: $F_a = 7.32 \cdot 10^{-3} \text{ N}$

$F_s = \sigma \cdot 2L = \sigma \cdot 2 \times 4a$ (2: two new surfaces)

When F_a is slightly greater than the surface tension force F_s , there will be

separation. $F_a = F_s \rightarrow \sigma = \frac{F_a}{8a} = \frac{7.32 \cdot 10^{-3}}{8 \cdot 5 \cdot 10^{-2}} = 85 \cdot 10^{-3} \text{ N/m}$

Exercise 2

1) $\Delta p = \frac{4\sigma}{r} = \frac{4 \cdot 25 \cdot 10^{-3}}{4 \cdot 10^{-2}} = 3.33 \text{ N/m}$

2) $\Delta W = \sigma \Delta S = \sigma \cdot 2(4\pi r^2) = 8\pi r^2 = 8\pi \cdot 25 \cdot 10^{-3} \cdot (3 \cdot 10^{-2})^2$

$\Delta W = 5.652 \cdot 10^{-4} \text{ J}$

Exercise 3

Jurin's law $\sigma \cos(\theta) = \frac{hr\rho g}{2}$

In the case of good wetting $\theta = 0$

1- $\sigma = \frac{hr\rho g}{2} = 38,62 \cdot 10^{-3} \text{ N/m}$

2- $h = \frac{4\sigma \cos\theta}{d\rho g} = \frac{4 \cdot 38,62 \cdot 10^{-3} \cdot \cos(155)}{10^{-3} \cdot 13600 \cdot 9,81} = -0.0106 \text{ m} = -10.60 \text{ mm}$

Exercise 4

For water $h_0 = \frac{2\sigma_0}{\rho_0 g r}$

For benzen $h = \frac{2\sigma}{\rho g r} \rightarrow \sigma = \sigma_0 \frac{h}{\rho_0} \frac{\rho_0}{\rho} = 29,375 \cdot 10^{-3} \text{ N/m}$

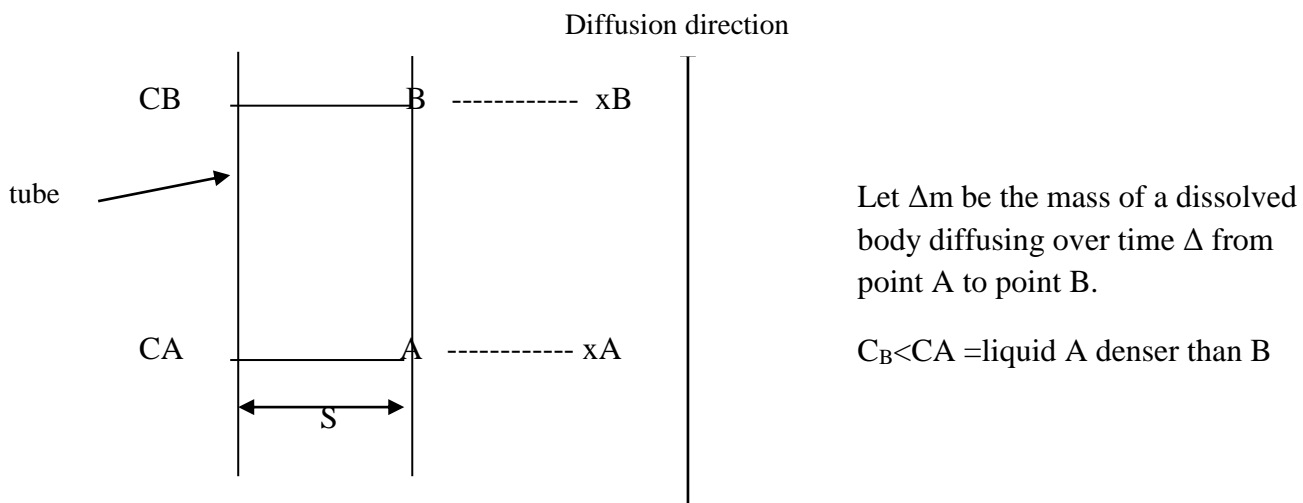
1- Diffusion :

If we mix 2 different miscible liquids (of different colors if possible), after a very short time, we notice that the mixture becomes homogeneous (liquids mix) for 3 reasons:

- a) Mechanical agitation, vibrations, etc.
- b) Temperature differences between the 2 liquids, which cause convection (molecules in the hot fluid are replaced by new molecules at lower temperatures).
- c) Thermal agitation of the molecules.

The phenomenon of free migration of molecules among others, i.e., these molecules spread homogeneously throughout the space available to them.

1-1 Fick's formula: the law governing diffusion



S : Pipe cross-section

Figure 4.1

Fick's law can be written as :

$$\Delta m = -D \frac{C_B - C_A}{x_B - x_A} \Delta t S$$

If A and B are very close, we have a differential

$$\frac{\Delta m}{\Delta t} = -D \frac{\Delta C}{\Delta x} S \Rightarrow \Delta m = -D S \frac{\Delta C}{\Delta x} \Delta t = -D \frac{\Delta C}{\Delta x} S \Delta t$$

$\frac{\Delta C}{\Delta x}$: Negative concentration gradient ($\Delta C < 0$, $\Delta x > 0$)

S: Tube cross – section or diffusion surface (m^2 ou cm^2)

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D : Diffusion coefficient that does not depend on the nature of the solvent or the nature of the solute. Unit of diffusion coefficient ($\frac{m^2}{s}$ ou bien $\frac{cm^2}{s}$)

$\frac{\Delta m}{\Delta t}$: Diffusion rate or mass of solute that has moved over time Δt from point x_A of weight concentration C_A to point x_B of weight concentration C_B .

Alternatively, we can have the number of diffusing moles (instead of the mass of molecules) per unit time $\frac{\Delta n}{\Delta t}$ (Φ : *diffusion molar flow*) $\frac{\Delta n}{\Delta t} = -D S \frac{\Delta c^M}{\Delta x}$

The diffusion coefficient D depends on the experimental diffusion conditions temperature, nature of the solute and nature of the solvent.

$$D = \frac{k_B T}{f}$$

Where k_B is Boltzmann's constant ($1.38 \cdot 10^{-23} \text{ J K}^{-1}$), T: Temperature in K°

f: coefficient of friction which is defined as follows; if the particle moves at speed v in the liquid medium, it must overcome a friction force F proportional to v $F = f v$

In the particular case where the particle is spherical of radius r, we can show (Stokes' law) that $F = 6 \pi \eta r v$ i.e. that $f = 6 \pi \eta r$

D is proportional to temperature T. If both temperature and diffusion coefficient increase, then $\frac{\Delta n}{\Delta t}$ increases.

Consider two solutes with radii r_1 and r_2 diffusing in two media with viscosities η_1 and η_2 and temperatures T_1 and T_2 respectively.

We have:

$$D_1 = \frac{k T_1}{6 \pi \eta_1 r_1}, \quad D_2 = \frac{k T_2}{6 \pi \eta_2 r_2} \quad \text{so} \quad \frac{D_2}{D_1} = \frac{T_2}{T_1} \frac{\eta_2}{\eta_1} \frac{r_1}{r_2}$$

$$\Rightarrow D_2 = \frac{T_2}{T_1} \frac{\eta_2}{\eta_1} \frac{r_1}{r_2} D_1$$

We take the case of two solutes 1 and 2 which are different but diffuse under the same experimental conditions. ($\eta_1 = \eta_2$, $T_1 = T_2$) So

$$D_2 = \frac{r_1}{r_2} D_1$$

In this particular case, let's assume that the volume density of the solute1 molecules is equal to that of the 2nd solute. ($\rho_1 \approx \rho_2$).

We have $\rho = \frac{m}{V}$ avec $V = \frac{4}{3} \pi r^3$ (V : volume of a spherical molecule)

$$\text{With } M = N m \implies \rho = \frac{m}{V} = \frac{m}{\frac{4}{3} \pi r^3} \implies r = \sqrt[3]{\frac{M}{\frac{4}{3} \pi N \rho}}$$

We then obtain for both solutes $\frac{r_1}{r_2} = \sqrt[3]{\frac{M_1}{M_2}}$ because ($\rho_1 \approx \rho_2$).

For two solutes of molar masses M_1 and M_2 diffusing under the same experimental conditions, the diffusion coefficient D_2 is given by this formula :

$$D_2 = \sqrt[3]{\frac{M_1}{M_2}} D_1$$

1-2 Complete mathematical expression of Fick's law (Fick's 2nd law) :

We need to replace Δm and Δt by differentials, in other words ratios by derivatives.:

$$dm = -D \frac{\partial C}{\partial x} S dt \implies dm = J S dt \quad \text{with } J = -D \frac{\partial C}{\partial x}$$

We use ∂ for the concentration gradient, as this is a partial derivative, C being a function of 2 variables x et t.

Let the volume element $dV = S dx$

The number of moles entering during dt

$$dn_1 = J_x S dt$$

The number of moles leaving during dt

$$dn_2 = J_{x+dx} S dt$$

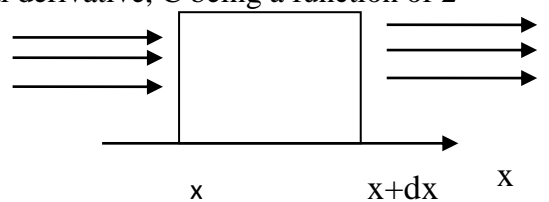


Figure 4.2

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The number of moles contained in the volume element dV is dn $dn = dn_1 - dn_2 = dC S dx = J_x S dt - J_{x+dx} S dt$

$$dn = -(J_{x+dx} S dt - J_x S dt) = -dJ S dt \quad \text{with } c = \frac{n}{V}$$

we obtain $dC dx = -dJ dt \implies \frac{dC}{dt} = -\frac{dJ}{dx}$

$$\implies \frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x} \quad \text{avec } J = -D \frac{\partial C}{\partial x}$$

$$\implies \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad \text{Fick's 2nd law}$$

In the particular case where $\frac{\partial C}{\partial t} = 0$ (steady state)

Fick's 2nd law implies that $\frac{\partial^2 C}{\partial x^2} = 0 \implies C(x) = A x + B$

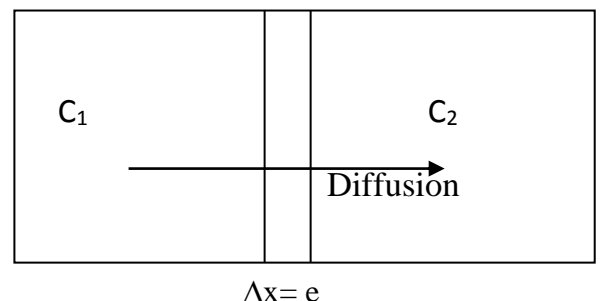
Where A and B are constants. In the particular case where the regime is stationary, the variation C with x is a straight line.

1-3 Diffusion through a membrane:

We consider that $C_1 > C_2$ avec

$$C(0) = C_1$$

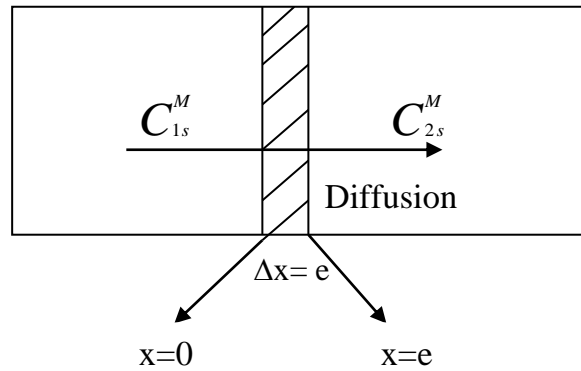
$$C(e) = C_2$$



Consider a membrane with surface area S and thickness e, traversed by pores whose total surface area is denoted by S_p . This membrane separates two media whose molar concentrations are respectively C_{1s}^M et C_{2s}^M .

The membrane allows solute molecules to diffuse. Assuming that $C_{1s}^M > C_{2s}^M$ distribution is from 1 to 2.

We propose to find the form of the concentration inside the membrane. If we assume that the diffusion regime is stationary and that the concentrations C_{1s}^M et C_{2s}^M are kept constant on both sides of the membrane during diffusion.



We are looking for the shape of C^M inside the membrane, considering that the regime is stationary.:

$$\frac{\partial C^M}{\partial t} = 0 \quad -1-$$

$$\text{Fick's 2nd law: } \frac{\partial C^M}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad -2- \implies \frac{\partial^2 C}{\partial x^2} = 0$$

$$\implies C(x) = A x + B \quad -3-$$

Using the boundary conditions:

$$x=0, \quad C^M(x) = C^M(0) = C_1^M$$

$$x=e, \quad C^M(x) = C^M(e) = C_2^M$$

Using these boundary conditions, we obtain the constants A and B.

$$A = \frac{C_2^M - C_1^M}{e}, \quad B = C_1^M$$

Replacing A and B in equation 3 gives us:

$$C^M(x) = \frac{C_2^M - C_1^M}{e} x + C_1^M$$

A is expressed in mol/m⁴ and its value is negative, B is expressed in mol/m³

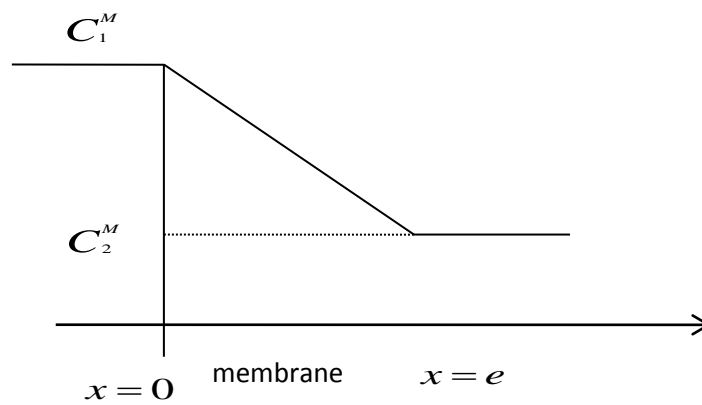


Figure 4.3 : Variation in concentration inside the membrane
inside the membrane $C(x)$ is a negative slope line.

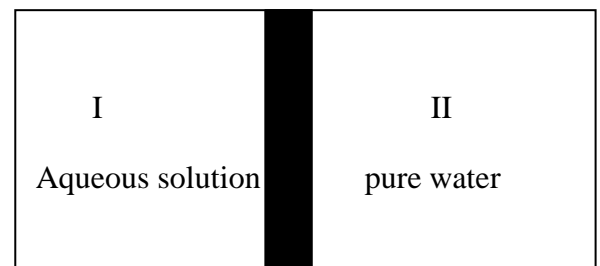
2- Osmosis and osmotic pressure:

2-1 Définition :

Two compartments separated by a selective hemi-permeable membrane Figure 4.4, permeable only to water..

Two compartments separated by a selective hemi-permeable membrane Figure 4.4, permeable only to water.

- Compartment I contains an aqueous
aqueous solution of a body that cannot pass through
the membrane.



- Compartment II contains pure water.

Figure 4.4

The system is therefore not in equilibrium, as all the water diffuses from II to I (compartment II is completely empty).

This passage of pure solvent through a hemipermeable membrane is called osmosis, from the less concentrated solution to the more concentrated one.

2-2 Osmotic pressure

To measure the pressure causing the water flow, the experiment is carried out in the Dutrochet osmometer (Fig. 4.5) or the following set-up:

The ascent of the liquid in the compartment containing the solution generates an overpressure Hydrostatic overpressure, which opposes the pressure which causes the water to diffuse.

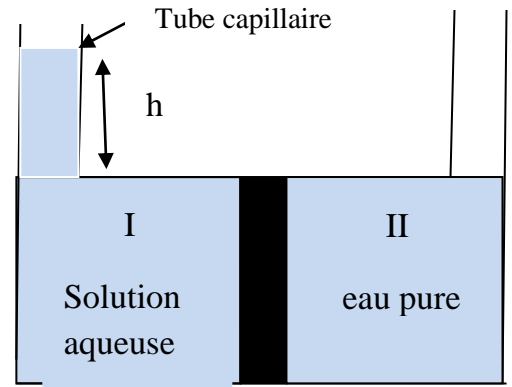


Figure 4.5

At equilibrium, the hydrostatic pressure that balances the diffusion of water into the solution is called osmotic pressure. In other words, the flow of solvent produces an imbalance. So, to achieve equilibrium, sufficient hydrostatic pressure must be exerted on the solution.

Definition: the osmotic pressure of a solution is the pressure that must be exerted on the solution to prevent its pure solvent from crossing the membrane separating the 2 phases.

It is given by van 't Hoff's law

$$\pi = \rho g h = C^0 R T$$

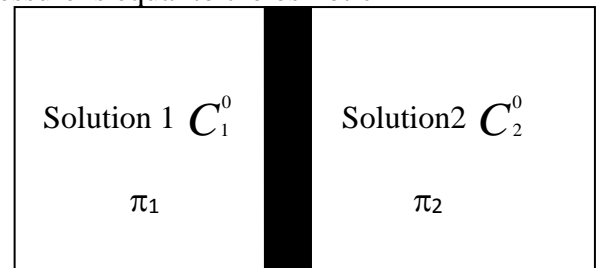
- h: height difference (m), g: gravity = 9.81 m/s², ρ: density, R: perfect gas constant = 8.314 J K⁻¹ mol⁻¹, T: absolute temperature in Kelvin, C⁰: osmolar concentration = i C_M (osmole/l) and π osmotic pressure pascals (N/m²).
- In the case of two aqueous solutions opposed to pure water across a hemipermeable membrane, the osmotic pressure is equal to the sum of the osmotic pressures π = π₁ + π₂

$$\pi = R T (C_1^0 + C_2^0)$$

If two solutions of concentrations C_1 and C_2 are separated by a hemipermeable membrane, the solvent will diffuse from the less concentrated hypotonic solution to the more concentrated hypertonic solution. In this case, osmotic pressure is equal to the osmotic

pressure difference
$$\pi = RT(C_1^0 - C_2^0)$$

Two solutions are iso-osmotic if, compared to a perfect hemipermeable membrane is opposed to a pure solvent, they have the same osmotic pressure, therefore the same osmolar concentration..



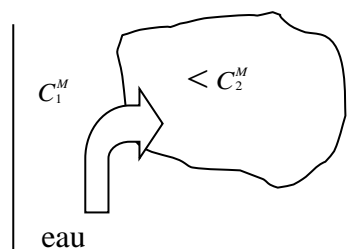
Sens de diffusion de solvant Avec $C_1^0 > C_2^0$

$$\pi_1 = \pi_2 \Leftrightarrow RT C_1^0 = RT C_2^0 \Leftrightarrow C_1^0 = C_2^0$$

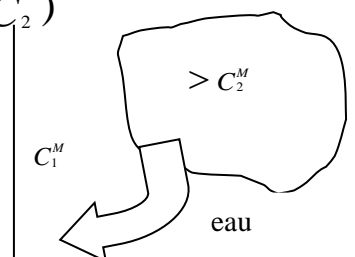
Iso-osmolar and hemipermeable membrane \Leftrightarrow iso-osmotic same osmotic pressure.

2-3 Biological applications:

- Cell membranes (red blood cells) separate the extracellular and intracellular media.
- - If the external medium is hypotonic (less concentrated), water penetrates the cells, causing them to swell - a phenomenon known as turgor.. $C_1^M < C_2^M$



- If the external environment is hypertonic (more concentrated), the cell loses all its water and shrinks (plasmolysis). $C_1^M > C_2^M$



3-Permeability :

The membrane's porosity is defined as the ratio of the total surface area of Sp pores to its surface area.

$$\varepsilon_0 = \frac{S_p}{S} < 1$$

Fick's first law is expressed as a function of S and S_p

$$\frac{dn}{dt} = -D \varepsilon_0 S \frac{C_2^M - C_1^M}{e} \quad \Longrightarrow \quad \frac{dn}{dt} = -D \frac{\varepsilon_0 S}{e} \Delta C^M$$

$\Delta C^M = C_2^M - C_1^M$, the quantity $\frac{\varepsilon_0 S}{e}$ is called the permeability of the membrane and is denoted by P_e, which is the same as velocity. Fick's 1st law can therefore be written as a

function of permeability as follows $\frac{dn}{dt} = -P_e S \Delta C^M$

Exercices :

Exercice 1 :

1- Show, for spherical particles, that the diffusion coefficient is inversely proportional to the cube root of the molar mass?

2- The diffusion coefficient of lactose is $4.8 \cdot 10^{-6} \text{ cm}^2 \cdot \text{s}^{-1}$ and its molar mass is 342 g. Calculate the diffusion coefficient of urea if its molar mass is 60 g.

3- To purify urea in a two-compartment system separated by a membrane, the diameter of the molecules must be less than the diameter of the membrane pores. Calculate its minimum value. (Knowing that $\eta = 10^{-2}$ poise and $T = 37^\circ \text{ C}$)

4- A hemoglobin solution with a concentration of $3 \cdot 10^{-4} \text{ mol/l}$ diffuses through a membrane with a diffusing surface area $S = 600 \text{ mm}^2$ to a concentration of $7 \cdot 10^{-5} \text{ mol/l}$. Given $D_{\text{hemoglobin}} = 6.9 \cdot 10^{-7} \text{ cm}^2/\text{s}$ and $M_{\text{hemoglobin}} = 68 \text{ Kg/mol}$.

Determine the mass of hemoglobin in micrograms that has moved 2 cm over 5 minutes.

Boltzmann's constant $k_B = 1,38 \cdot 10^{-23} \text{ J/K}$

Exercice 2 :

Consider a porous membrane separating two compartments containing sucrose at concentrations of 0.5 and 0.2 mol/l. These concentrations are kept constant as the sucrose molecules diffuse across the membrane. Steady state is assumed.

- What is the direction of the diffusion current density J ?
- Find the sucrose diffusion coefficient.
- Establish the law of variation of the concentration inside the membrane.
- What is the value of the sucrose current density at 25°C ?
- What would be the molar flow of sucrose at 0°C ?

We give: Molar flow of sucrose $\phi_s = 1.2 \cdot 10^{-2} \text{ mol} \cdot \text{s}^{-1}$ at 25°C .

Total membrane pore surface $S_p = 0.05 \text{ m}^2$.

Membrane thickness $h = 10 \mu\text{m}$

Personal work

Exercice 1 :

At 25°C, the mobilities of anions and cations in a NH_4NO_3 solution that dissociates completely into NH_4^+ and NO_3^- ions are respectively $\mu_+ = 7,6 \cdot 10^{-8} \text{ m}^2\text{s}^{-1}\text{V}^{-1}$ et $\mu_- = - 7,4 \cdot 10^{-8} \text{ m}^2\text{s}^{-1}\text{V}^{-1}$

- 1- Calculate the friction and diffusion coefficients of the cations and anions?
- 2- Assuming these two ions are spherical, calculate the radii of the cations and anions?

Exercice2:

A container is separated by a diffusing membrane (surface area $S=200\text{cm}^2$ and thickness 0.12mm) into two compartments of volume $V_1= 1.6\text{l}$ and $V_2= 1.4 \text{ l}$ containing 0.5mol urea and 0.8mol glucose respectively.

1- Determine the initial molar flux of urea and glucose in mol/s. Knowing that the diffusion coefficient of urea at the membrane is $D_u= 10^{-9} \text{ m}^2/\text{s}$,

We give $M_{\text{urea}}=60 \text{ g/mol}$ and $M_{\text{glucose}}= 180 \text{ g/mol}$.

- 2- What is the mass concentration of urea at equilibrium?

Answers:

Exercise 1 :

$$1- D = \frac{cst}{\sqrt[3]{M}}$$

$$D = \frac{kT}{f}, \text{ with } f = 6\pi\eta r \longrightarrow D = \frac{kT}{6\pi\eta r} \quad -1-$$

$$\rho = \frac{m}{V}; \quad M = Nm; \quad V = \frac{4\pi}{3} r^3$$

$$m = \rho V = \rho \frac{4\pi}{3} r^3 \longrightarrow \frac{M}{N} = \rho V = \rho \frac{4\pi}{3} r^3$$

$$\longrightarrow r = \sqrt[3]{\frac{3M}{N\rho 4\pi}} \quad -2-$$

By replacing -2- in -1- we obtain

$$D = \frac{kT}{6\pi\eta} \sqrt{\frac{N\rho 4\pi}{3}} \frac{1}{\sqrt[3]{M}}$$

$$\text{So } cst = \frac{kT}{6\pi\eta} \sqrt{\frac{N\rho 4\pi}{3}} \longrightarrow D = \frac{cst}{\sqrt[3]{M}}$$

$$2- \frac{D_1}{D_2} = \sqrt[3]{\frac{M_2}{M_1}} \longrightarrow D_2 = D_1 \sqrt[3]{\frac{M_2}{M_1}}$$

$$D_2 = 4,8 \cdot 10^{-6} \sqrt[3]{\frac{342}{60}} = 8,57 \cdot 10^{-6} \text{ cm}^2 / \text{s}$$

3- The minimum membrane pore size is the diameter of the molecules.

$$D_{urea} = \frac{kT}{6\pi\eta r_{urée}} \longrightarrow r_{urée} = \frac{kT}{6\pi\eta D_{urea}} = \frac{1,38 \cdot 10^{-23} \cdot 310}{6\pi \cdot 10^{-3} \cdot 8,57 \cdot 10^{-10}}$$

$$r_{urea} = 2,64 \cdot 10^{-10} \text{ m}$$

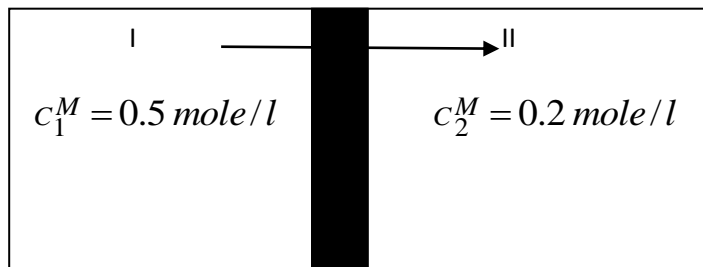
$$\text{pore diameters} = 2 r_{urea} = 5,29 \cdot 10^{-10} \text{ m}$$

$$3- \frac{\Delta m}{\Delta t} = -D S \frac{\Delta C}{\Delta x} = -D S \frac{C_2^p - C_1^p}{\Delta x}$$

$$\Delta m = \frac{-6.9 \cdot 10^{-7} \cdot 10^{-4} \cdot 600 \cdot 10^{-6} \cdot (7 \cdot 10^{-5} - 3 \cdot 10^{-4}) \cdot 10^3 \cdot 68}{2 \cdot 10^{-2} \cdot 5 \cdot 60} = 9,71 \cdot 10^{-9} \text{ kg}$$

$$\Delta m = 9,71 \mu\text{g}$$

Exercice 2 :



1- 1- Direction of diffusion from I to II from the most concentrated to the least concentrated

$$D_{sac} = \frac{k T}{6 \pi \eta r_{sac}}$$

$$\frac{\Delta n}{\Delta t} = -D_{sac} S_p \frac{\Delta C}{\Delta x} = -D_{sac} S_p \frac{(C_2 - C_1)}{h}$$

$$D_{sac} = -\left(\frac{\Delta n}{\Delta t}\right) \frac{h}{S_p (C_2 - C_1)} = \frac{1.2 \cdot 10^{-2} \cdot 10^{-5}}{0.05(0.2 - 0.5) \cdot 10^3}$$

$$D_{sac} = 0.08 \cdot 10^{-7} = 8 \cdot 10^{-9} \text{ m}^2 / \text{s}$$

The law of variation of concentration, working in a steady state $\frac{\partial C}{\partial t} = 0$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}, \text{ we therefore obtain } C(x) = A x + B$$

Using the boundary conditions $C(0) = C_1$ and $C(e) = C_2$, we find that $C(x)$ is written as follows:

$$C(x) = \frac{C_2 - C_1}{h} x + C_1$$

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$$C(x) = \frac{(0.2 - 0.5) 10^3}{10^{-5}} x + 0.5 10^3 \quad \text{mol/m}^3$$

$$C(x) = -3 \cdot 10^7 x + 0.5 \cdot 10^3$$

Current density:

$$J = -D_{sac} \frac{\partial C}{\partial x} = -0.08 \cdot 10^{-7} (-3 \cdot 10^7) = 0.24 \text{ mol / s m}$$

$$\Phi_n = -D S_p \frac{\Delta C}{\Delta x} = -\frac{k T}{6 \pi \eta r} S_p \frac{\Delta C}{\Delta x}$$

$$\frac{\Phi_1}{\Phi_2} = \frac{T_1}{T_2} \quad \longrightarrow \quad \Phi_2 = \Phi_1 \frac{T_1}{T_2} = 1,2 \cdot 10^{-2} \frac{(273)}{(273 + 25)} = 0.01099 \text{ mol / s}$$

Exercices :

Exercice 1 :

The upwelling of sap in trees is the result of osmotic pressure exerted by the difference in sugar concentration between the sap at root level and the water in the soil.

Knowing that 10 g of the mass of sap is in the form of sugar ($C_{12}H_{22}O_{11}$) in one liter of sap at a temperature of $27^{\circ}C$.

1- Calculate the concentration of sugar in the sap, expressed in moles/liter.

2- Calculate the osmotic pressure

3- The maximum height reached by the sap in these trees.

Given: C=12 , H= 1, O=16 (g/mol) and $R = 8.31 J.K^{-1}.mole^{-1}$.

Exercice 2 :

A semi-permeable membrane separates two solutions at $20^{\circ}C$. One contains 4 g of glucose, molar mass 180 g/mol, completely dissociated in 10 liters of water, and the other contains 9.5 g of $MgCl_2$, molar mass 95 g/mol, in the same volume of water, 50% dissociated.

- What is the physical phenomenon observed? Can you explain?

- What is the resulting osmotic pressure exerted on the membrane?

Personal work

Exercice 1 :

What is the height of ascent of an aqueous solution containing 10^{-4} mol/l of sodium proteinate $Na_{18}P$, with dissociation coefficient $\alpha=0.25$, in the tube of a Dutrochet osmometer immersed in a container containing pure water at $30^{\circ}C$ at equilibrium. (We'll assume that the density of the solution is equal to that of water and that the osmometer membrane is strictly hemipermeable).

Exercice 2:

The cell membrane of red blood cells is considered, for a short time, to be impermeable to NaCl and glucose, while it is permeable to urea. Given that the average intracellular osmolarity of red blood cells is $0.31 \text{ osmol l}^{-1}$, what can be expected if red blood cells are immersed in each of the following 7 aqueous solutions?

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NaCl	Glucose	urea
(1) 9.1 g/l	(2) 55.8 g/l	(7) 18.6 g/l
(3) 7.3 g/l	(4) 45 g/l	
(5) 11.7 g/l	(6) 72 g/l	

$$M_{\text{NaCl}} = 58.5 \text{ g mol}^{-1}, M_{\text{glucose}}=180 \text{ g mol}^{-1}, M_{\text{urea}}= 60 \text{ g mol}^{-1}$$

Answer:

Exercise 1 :

$$c^M = \frac{n}{V} = \frac{m}{M V} = \frac{10}{(342)1} = 0.029 \text{ mole/l}$$

$$\pi = R T C^0$$

$$\pi = 8.314 \text{ 300 0.029 } 10^3 = 7.292 \cdot 10^4 \text{ pas}$$

$$\pi = \rho g h \Rightarrow h = \frac{\pi}{\rho g} = \frac{7.292 \cdot 10^4}{10^3 \cdot 9.81} = 7.43 \text{ m}$$

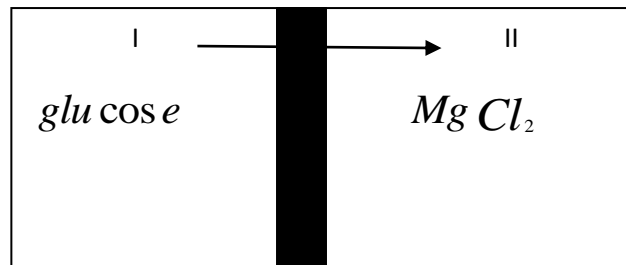
Exercise 2 :

The phenomenon observed is osmosis

because the membrane is selective, allowing

only the passage of water molecules.

molecules.



$$\pi = R T (C_1^0 - C_2^0)$$

$$C_1^M = \frac{m_1}{M V} = \frac{4}{180 \cdot 10} = 0.0022 \text{ mole/l}$$

$$C_2^M = \frac{m_2}{M V} = \frac{9.5}{95 \cdot 10} = 0.01 \text{ mole/l}$$

Glucose will not dissociate so $\alpha=0$ and therefore :

$$C_1^0 = C_1^M = 0.0022 \text{ osmole/l}$$

$$C_2^0 = i C_2^M = (1 + \alpha(v - 1)) C_2^M \quad \Rightarrow$$

$$C_2^0 = i C_2^M = (1 + 0.5(3 - 1)) 0.01 = 0.02 \text{ osmole/l}$$

$$\pi = 8.314 \cdot 293 (0.02 - 0.0022) \cdot 10^3 = 4.3360 \cdot 10^4 \text{ pas}$$

Chapter V: Study of viscosity :

1- Laminar and turbulent flows :

A liquid is the seat of internal friction. If we set a region of a liquid in motion, we see 2 things:

- a- This movement spreads from close to close to neighboring regions.
- b- It progressively weakens if you stop moving it. This liquid friction is due to Van der Waals forces of attraction (molecular attraction), which oppose the relative movements of neighboring molecules.

This internal fluid friction is called viscosity.

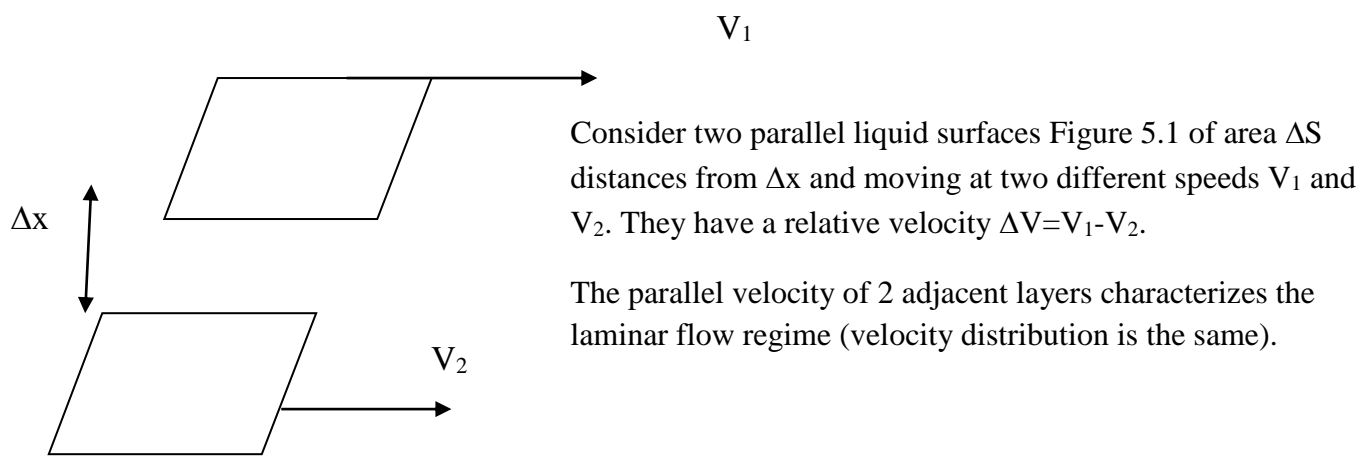


Figure 5.1

The viscosity strength is :

$$\Delta f = \eta \Delta S \frac{\Delta V}{\Delta x} \quad , \quad \frac{\Delta V}{\Delta x} \text{ is the velocity gradient}$$

η : the liquid's viscosity coefficient.

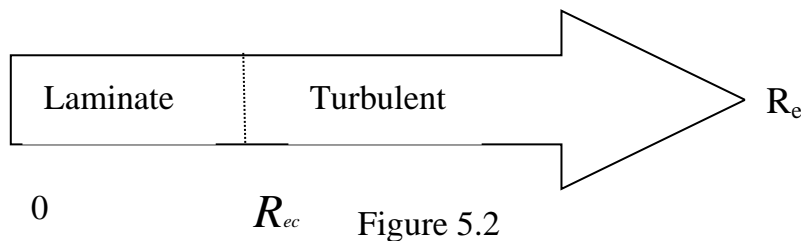
The presence of viscosity therefore results in a pressure difference between the ends of the tube when a fluid is flowing smoothly through a horizontal tube:

In other words, there's a pressure drop. Pressure decreases in proportion to the distance covered by the fluid. During its flow, a fluid can present two flow regimes: laminar flow or turbulent flow. The transition from laminar to turbulent flow depends on the Reynolds number Re .

$$Re = \rho \frac{v D}{\eta}$$

Re is a dimensionless number, ρ : fluid density in kg/m^3 , v : flow velocity in m s^{-1} ,
 D : pipe diameter in m and η :fluid viscosity in $\text{kg m}^{-1}\text{s}^{-1}$

La transition du régime laminaire au régime turbulent s'observe Figure 5.2 pour $R_e \approx R_{ec} = 2000$



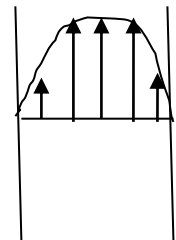
If we take a sufficiently narrow cylindrical tube (capillary tube), the only possible flow regime is laminar, as the layers of fluid slide over each other without mixing. The flow velocity has a parabolic distribution, is zero on the walls and passes through a maximum at the center of the tube.

The volume flow (volume per second) (m³/s) in this tube is given by Poiseuille's formula

$$Q_v = \frac{\pi}{8\eta} \frac{\Delta P}{L} R^4 \quad \text{Laminar, linear or Poiseuille regime)}$$

L: tube length, ΔP : pressure difference between tube inlet and outlet

R: tube radius



The average speed is $\bar{V} = \frac{\Delta P R^2}{8 \eta L}$

$\frac{\Delta P}{L}$: Represents pressure drop per unit length

2- Flow resistance and viscosity measurements

a- Resistance to flow :

Resistance to flow is defined by the quantity R :

$$R = \frac{8 \eta L}{\pi R^4} = \frac{128 \eta L}{\pi D^4} \quad \text{Avec } D = 2 R \quad \text{et} \quad R = \frac{\Delta P}{Q_v}$$

In the case of a series duct system, the flow resistances add up to

$$R_T = \sum_{i=1}^n R_i = R_1 + R_2 + \dots$$

In the case of a parallel flow system, the total flow resistance is given by :

$$\frac{1}{R_T} = \sum_{i=1}^n \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Application: pulmonary circulation model:

- Blood arrives through the right heart, passes through the capillaries, in contact with the pulmonary alveoli, and exits through the left heart Figure 5.3.

The circulation is made up of different networks:

arterial network: \longrightarrow arterioles \longrightarrow capillaries

The venous network: \longrightarrow capillaries \longrightarrow veins.



Figure 5.3

a- Viscosity measurements :

There are several methods for measuring viscosity, including :

- Ostwald viscosimeter (Poiseuille's law)
- Hoppler viscosimeter (Stokes' law)
- Covelle rotating cylinder.

The Hoppler viscometer is described in detail below:

A solid sphere is dropped, in free fall, between 2 divisions of the apparatus filled with a liquid of known viscosity η_0 et the time of fall t_0 the distance travelled $x_0=v_0 t_0$ (measured) -1-

Repeat the operation with the same sphere in another liquid of unknown viscosity η_1 which travels the same distance $x_1=v_1 t_1$ such that $x_1=x_0$ -2-

$$-1- = -2- \implies v_1 t_1 = v_0 t_0 \implies \frac{t_0}{t_1} = \frac{v_1}{v_0} \quad -3-$$

Using Stokes' law:

$$v_1 = \frac{2}{9} r^2 (\rho_2 - \rho_1) \frac{g}{\eta_1}$$

$$v_0 = \frac{2}{9} r^2 (\rho_2 - \rho_0) \frac{g}{\eta_0}$$

ρ_2 : density of solid, ρ_1 : density of liquid 1 and ρ_0 : density of liquid 0.

$$\implies \frac{v_1}{v_0} = \frac{(\rho_2 - \rho_1) \eta_0}{(\rho_2 - \rho_0) \eta_1} \quad -4-$$

$$-3- = -4- \implies \frac{t_0}{t_1} = \frac{v_1}{v_0} = \frac{(\rho_2 - \rho_1) \eta_0}{(\rho_2 - \rho_0) \eta_1}$$

We deduce the viscosity η_1 of the liquid

3- Stokes' law

A spherical solid of radius r moving at speed v in a fluid encounters resistance from the liquid due to the viscosity of the medium η . This force, which opposes the force of motion of the solid, is :

$$f = 6 \pi \eta r V \quad (\text{Stokes' law}).$$

If the driving force \vec{F}_m is gravity corrected for buoyancy, noted as \vec{F}_a . The driving force is given by this equation : $\vec{F}_m = \vec{P} + \vec{F}_a$. Falling speed becomes constant when this driving force is balanced by the force of friction.

$$\Sigma \vec{F} = \vec{0} \Leftrightarrow \vec{F}_a + \vec{f} + \vec{P} = \vec{0} \Rightarrow \vec{P} = -(\vec{F}_a + \vec{f})$$

$$\Leftrightarrow |P| = |F_a + f|$$

$$\Leftrightarrow m g = F_a + 6\pi \eta r V$$

$$\rho_1 \frac{4}{3} \pi r^3 g = \rho_2 \frac{4}{3} \pi r^3 g + 6\pi \eta r V$$

$$V = \frac{2}{9} r^2 (\rho_1 - \rho_2) \frac{g}{\eta} \quad \text{Stokes' law (falling speed)}$$

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Falling speed therefore varies as the square of the radius. Larger grains sediment, and the velocity is negligible if the grains are small enough.

3-1 Application: measuring sedimentation velocity

Measurement of sedimentation velocity when particles (red blood cells) are subjected to gravity g followed by centrifugal acceleration γ .

Blood can be considered as a suspension of red blood cells in plasma with viscosity $\eta=1.46 \cdot 10^{-3}$ pas s, subjected to gravity g .

Red blood cells are considered spherical, with an average radius $r=2.7 \mu\text{m}$. The density of the red blood cells $\rho_0= 1100 \text{ kg/m}^3$, The density of the plasma $\rho_1= 1020 \text{ kg/m}^3$, $g=9.8 \text{ m/s}^2$

Using Stokes' formula, we obtain the sedimentation rate of these red blood cells:

$$V = \frac{2}{9} r^2 (\rho_0 - \rho_1) \frac{g}{\eta}$$

$$V = \frac{2}{9} (2.7 \cdot 10^{-6})^2 (1100 - 1020) \frac{9.8}{1.46 \cdot 10^{-3}} = 0.87 \mu\text{m s}^{-1}$$

i.e. a speed of approximately 3.14 mm/h

If the acceleration of gravity g is replaced by a centrifugal acceleration γ . This centrifugal force must be sufficient to neglect diffusion phenomena due to thermal agitation and the concentration gradient created.

Let's consider a molecule of mass m , located at distance x from the rotor rotating at constant angular velocity $w=2\pi N$.

The particle is subjected to :

$$\text{Centrifugal force } F_c = m_a \gamma = m_a w^2$$

- - Where m_a is the apparent mass of the particle in its medium, equal to its real mass minus the mass associated with the Archimedean dust..

$$m_a = m - m' = m \left(1 - \frac{m'}{m}\right) = m \left(1 - \frac{\rho_0}{\rho}\right)$$

$$F_c = m \left(1 - \frac{\rho_0}{\rho}\right) x w^2$$

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Friction force $F = f v$ where f : coefficient of friction .

Steady state is reached when the friction force balances the centrifugal force. $F_c = F$

$$m\left(1 - \frac{\rho_0}{\rho}\right) x \omega^2 = f v \longleftrightarrow \frac{m\left(1 - \frac{\rho_0}{\rho}\right)}{f} = \frac{v}{x \omega^2}$$

This ratio depends solely on the nature of the particle and the medium in which it is immersed, and defines the Svedberg constant S , expressed in seconds (s) or Svedbergs (S) as follows:

$$1S = 10^{-13} \text{ s.}$$

$$S \omega^2 x = v = \frac{dx}{dt}$$

The steady-state condition is

$$\int \frac{dx}{x} = S \omega^2 dt$$

$$\ln \frac{x_2}{x_1} = S \omega^2 (t_2 - t_1)$$

During the Δt ultracentrifugation time, the abscissa of particle x varies from x_1 to x_2 .

$$S = \frac{m\left(1 - \frac{\rho_0}{\rho}\right)}{f} \quad \text{avec} \quad f = \frac{k T}{D} = \frac{R T}{N D}$$

where D is the diffusion coefficient of the molecule and N is the avogadro number. Knowing that the molar mass is given by this equation $M = N m$. By replacing the expression of m in the equation for Svedberg's constant

we obtain :

$$M = \frac{R T}{\left(1 - \frac{\rho_0}{\rho}\right)} \frac{S}{D}$$

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Experimental measurement of S by ultracentrifugation and knowledge of the diffusion coefficient D enable us to determine the molecular molar mass of the macromolecule.

Exercices :

Exercice 1 :

A viscous Newtonian liquid with a dynamic viscosity coefficient $\eta = 1 \text{ mPa}\cdot\text{s}$ and a density of $1000 \text{ kg}\cdot\text{m}^{-3}$ flows through a 1cm-diameter pipe. What is the flow rate in litres/min required to change from laminar to turbulent flow?

Exercice 2:

In the aorta of a normal subject, the systolic velocity v_s is twice the mean velocity v , the diastolic velocity v_d is half the mean velocity. Determine the flow regimes in the aorta at the start of diastole and systole.

Given: radius of the aorta: 1cm and volume flow $Q_v = 5.4$ liters/min.

Exercice 3:

In a viscometer, a certain volume of water flows in 1 min; the same volume of a patient's blood flows in 3 min 20 sec.

Determine the relative viscosity of the patient's blood of density $1050 \text{ kg}/\text{m}^3$.

Exercice 4 :

A spherical particle of radius $r = 30 \text{ nm}$, density $d = 1.3$ is dropped without initial velocity into a medium of viscosity $\eta = 0.2 \text{ mPa}\cdot\text{s}$ and density $\rho_0 = 1 \text{ g}/\text{cm}^3$.

Calculate the diffusion coefficient D at $T = 27^\circ\text{C}$.

Establish the expression for the limiting sedimentation velocity. Calculate its value.

Exercice 5:

Using an ultracentrifuge with angular velocity w , at 293 K, determine the Svedberg constant of a protein A in water $S_A = 4.4 \text{ Sv}$.

Establish the two expressions for the Svedberg constant.

Establish the expression for the molar mass. Calculate the value of MA knowing that the density $\rho_A = 1330 \text{ kg}/\text{m}^3$ and the diffusion coefficient $DA = 63 \cdot 10^{-12} \text{ m}^2/\text{s}$.

Under the same experimental conditions, a second protein B is centrifuged, with sedimentation constant $S_B = 8.8 \text{ Sv}$ and density ρ_B close to ρ_A .

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Determine the molar mass of protein B.

Exercise 6 :

A globular particle dropped without initial velocity into a medium of viscosity $\eta = 1 \text{ mPa}\cdot\text{s}$ and density $\rho_0 = 1 \text{ g/cm}^3$, almost instantaneously reaches its limiting velocity v_L .

Subjected to ultracentrifugation at 60000 rpm, and when it is $x = 10 \text{ cm}$ from the axis of rotation, its speed is equal to v .

Calculate the ratio v/v_L at this distance from the axis.

Calculate the distance covered by this particle in 1 hour, given that its Svedberg constant is equal to 14 Sv

Answers :

Exercise 1 :

$$R = \frac{\rho \bar{v} D}{\eta} \text{ with } \bar{v} = \frac{Q_v}{s} \text{ Hence } Q_v = \frac{\eta \pi}{2 \rho} \times R \times r = 2.61 \cdot 10^{-4} \text{ l/min}$$

Exercise 2 :

$$\text{Average speed: } v = \frac{Q_v}{s} = \frac{5.4 \cdot 10^{-3}}{60 \pi (10^{-2})^2} = 0,287 \text{ m/s}$$

$$\text{In systole } v_s = 2v = 0,574 \text{ m/s}$$

$$\text{In diastole } v_d = v/2 = 0,143 \text{ m/s}$$

Critical flow velocity

$$\therefore v_c = \frac{\eta R}{\rho D} = 0,419 \text{ m/s}$$

In systole, therefore, the regime is turbulent; in diastole, it is laminar.

Exercise 3:

Let's call V the volume of water (index 1) or blood (index 2) flowing out.

$$V = (Q_v \times t)_{\text{eau}} = (Q_v \times t')_{\text{sang}}$$

With $D = \frac{\pi \times \Delta P \times r^4}{8\eta L}$ et $D' = \frac{\pi \times \Delta P' \times r^4}{8\eta' L}$ hence

$$\frac{Q_v \times \eta}{\Delta P} = \frac{Q_v' \times \eta'}{\Delta P'}$$

Since ΔP is proportional to a ρ , we deduce:

$$\frac{Q_v}{Q_v'} = \frac{\rho}{\rho'} \cdot \frac{\eta'}{\eta} \cdot \frac{t'}{t} \quad \text{soit} \quad \frac{\eta}{\eta'} = \frac{t'}{t} \cdot \frac{\rho'}{\rho} = \frac{200}{60} \times 1,050 = 3.5$$

Exercice 4 :

$$1- D = \frac{kT}{f} = \frac{kT}{6\pi\eta r} = \frac{1.38 \cdot 10^{-23} (300)}{6\pi \cdot 0.2 \cdot 10^{-3} \cdot 30 \cdot 10^{-9}} = 3.66 \cdot 10^{-11} \text{ m}^2/\text{s}$$

$$2- v_L = \frac{2}{9} \frac{r^2}{\eta} g \rho_0 (d-1) = \frac{2}{9} \frac{(30 \cdot 10^{-9})^2}{0.2 \cdot 10^{-3}} \cdot 10 \cdot 10^3 (1.3-1) = 0.1 \frac{\text{m}}{\text{s}}$$

Exercice 5 :

$$1- S_v = \frac{(m - m_0)}{f} = \frac{v}{\gamma}$$

$$2- M_A = \frac{S_v R T}{D_A (1 - \frac{\rho_0}{\rho_A})}$$

$$M_A = \frac{4.4 \cdot 10^{-13} \cdot 8.314 \cdot 293}{63 \cdot 10^{-12} (1 - \frac{1000}{1330})} = 68.56 \text{ kg/mol}$$

$$3- M_B = M_A \left(\frac{S_B}{S_A} \right)^{3/2} = 139.91 \text{ kg/mol}$$

Exercice 6 :

$$1- \frac{v_L}{v} = \frac{g}{\gamma} = \frac{g}{x \omega^2}$$

$$\frac{v}{v_L} = \frac{x \omega^2}{g} = \frac{10 \cdot 10^{-2} (6283.185)^2}{10} = 394.784 \cdot 10^3$$

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$$\omega = 2 \pi N = 2 \pi \left(\frac{60000}{60} \right) = 6283.185 \text{ rd / s}$$

$$2- S_v = \frac{v}{x \omega^2}$$

$$v = S_v x \omega^2 = 14 \cdot 10^{-13} \cdot 10^{-2} (6283.185)^2 = 5.52 \cdot 10^{-6} \text{ m / s}$$

$$l = v t = 5.52 \cdot 10^{-6} \cdot 3600 = 0.0198 \text{ m}$$

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