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Worksheet of Directed Work in Atomic Physics

Exercice 1 : Theoretical Analysis of the Model

1. Derivation of Angular Momentum Quantization :

Starting from Bohr's stability condition $\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r}$ and the quantization condition $mvr = n\frac{h}{2\pi}$, demonstrate that the radius of the orbit is given by :

$$r_n = \frac{\epsilon_0 h^2 n^2}{\pi m_e e^2}$$

2. Total Energy :

Express the total energy E_n of an electron in a hydrogen-like ion of atomic number Z as a function of n and Z . Provide a physical justification for why the energy is negative.

3.) Wavelength and Frequency Calculations :

For the hydrogen atom :

- (a) Calculate the wavelength (in nm) and frequency (in Hz) of the photon emitted during the transition $n = 4 \rightarrow n = 2$.
- (b) Determine the region of the electromagnetic spectrum for this radiation.
- (c) Calculate the energy of this transition in joules and electronvolts.

4.) Absorption vs Emission :

A hydrogen atom in its ground state absorbs a photon of wavelength 97.3 nm.

- (a) Calculate the energy of the absorbed photon.
- (b) To which energy level is the electron excited?
- (c) What are the possible wavelengths of photons that could be emitted when the electron returns to the ground state?

5.) Series Limit Calculations :

- (a) Calculate the shortest wavelength in the Balmer series (transition $n = \infty \rightarrow n = 2$).
- (b) Calculate the longest wavelength in the Paschen series (transition $n = 4 \rightarrow n = 3$).
- (c) Determine which of these two photons is more energetic.

Exercice 2 : Application to Hydrogen-like Ions

The He^+ ion ($Z=2$) and Li^{2+} ion ($Z=3$) are important hydrogen-like systems in medical physics.

1. Energy Transitions :

Compare the energies of the photons emitted during the $n = 3 \rightarrow n = 2$ transition for :

- The hydrogen atom ($Z=1$)
- The He^+ ion ($Z=2$)
- The Li^{2+} ion ($Z=3$)

What pattern do you observe? Generalize for a hydrogen-like ion of charge Z .

2. Limit Wavelength :

For the He^+ ion, calculate the wavelength of the photon emitted during the transition $n = \infty \rightarrow n = 2$. In which region of the electromagnetic spectrum does it lie?

Exercise : In medicine, understanding matter-radiation interactions is fundamental, particularly for medical imaging (CT scanners, MRI) and radiotherapy. The Bohr model, although simplified, provides a foundational understanding of light emission and absorption by atoms.

Constants and Data

- Rydberg constant for hydrogen : $R_H = 1.097 \times 10^7 \text{ m}^{-1}$
- Planck's constant : $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$
- Speed of light in a vacuum : $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$
- $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
- $1 \text{ nm} = 10^{-9} \text{ m}$

Part 1 : Course Review Questions

- 1) State the two main postulates of the Bohr model for the hydrogen atom.
- 2) What is a **photon**? What is the relationship between its energy E and the wavelength λ of the associated radiation?
- 3) Define what an **energy level** is for an electron in an atom. What happens when the electron moves from a level n_i to a level n_f where $n_f < n_i$?

Part 2 : Numerical Application (The Balmer Series) The Balmer series corresponds to all electronic transitions where the electron falls to the energy level $n_f = 2$. This series produces lines in the visible region of the electromagnetic spectrum.

a) Calculating a wavelength :

An excited electron in a hydrogen atom relaxes by moving from the level $n_i = 3$ to the level $n_f = 2$.

- 1) Calculate the wavelength λ (in nm) of the photon emitted during this transition. Use the Rydberg formula :

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

- 2) Is this emitted radiation visible to the naked eye? Justify your answer. (The visible range extends approximately from 400 nm to 800 nm).

b) Calculating an energy :

- 1) Calculate the energy (in Joules, then in electronvolts) of the photon emitted during the transition $n_i = 3 \rightarrow n_f = 2$.
- 2) Compare this energy to that of the transition $n_i = 4 \rightarrow n_f = 2$. Which one is more energetic? Why is this logical?

Part 3 : Synthesis Problem (Link to Physiology) Hemoglobin, the protein that transports oxygen in the blood, has an active part called "heme". At the center of the heme is an iron atom (Fe^{2+}). For simplification purposes, we can model an electron of this ferrous ion as if it were in a "Bohr atom" where the energy of a level n is given by

$$E_n = -\frac{13.6}{n^2} \text{ eV}.$$

- a) Calculate the energies of the levels $n = 1$, $n = 2$, and $n = 3$ for this electron.
- b) When blood is well-oxygenated (arterial), the electron is in its ground state ($n = 1$). When the blood is low in oxygen (venous), this electron can absorb a photon with a wavelength $\lambda = 430 \text{ nm}$ and move to an excited state.
 - 1) Calculate the energy (in eV) of the photon with a wavelength of 430 nm.
 - 2) Considering the levels calculated in question 1, to which level n was this electron excited? Justify your reasoning.
 - 3) It is this absorption of light that gives deoxygenated blood its **bluish-violet** color (observable in veins under the skin), while oxygenated blood, which does not absorb in the visible range, is red. Explain this phenomenon in one sentence, linking it to the Bohr model.